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Research Paper

Dynamic Default Rates

Date: May 2008
Reference Number:8/4

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DYNAMIC DEFAULT RATES

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First version: August 2005
This version: May 2008*

Abstract

This paper develops new, dynamic and conditional versions of Vasicek's widely used single factor, default rate distribution. We employ our new class of distributions in modelling US bank loan losses. We analyze the implications for risk, capital, diversification and cyclical effects in loan portfolios and investigate how observed macroeconomic factors such as shocks to industrial production and unemployment affect the distribution of credit losses. A strength of our approach is the simplicity with which one may incorporate rich patterns of autocorrelation and dependence on observable factors into default rate distributions.

JEL classification: G33; E44; G21
Keywords: Default; Bank loan; Loss distribution, Capital, Stress test

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1 Introduction

Estimates of bond and loan loss distributions have important implications for (i) risk management decisions by banks, (ii) financial regulation design by supervisors, (iii) assessments of structured products by ratings agencies, and (iv) the pricing of CDOs and ABSs by investors. In this paper, we develop simple but highly tractable techniques for analysing loan and bond default rates.

The two most widely used approaches to modeling defaults are hazard rate models and latent variable models. Recent research analyzing loan and bond defaults using hazard rate models includes Duffie, Saita, and Wang (2007), Duffie, Eckner, Horel, and Saita (2006) and Das, Duffie, Kapadia, and Saita (2007). Recent studies following a latent variable approache include McNeil and Wendin (2007) and McNeil and Wendin (2006).

In the case of large and diversified portfolios, Vasicek (1991) suggested an attractive way to model loan losses. Building on a simple latent variable model of default, Vasicek shows that, as the number of exposures in the portfolio increases to infinity, the distribution of the loss rate (the fraction of obligors that default) converges to a simple closed form expression.

Vasicek's loan loss distribution has been widely employed by academic and industry researchers in modeling credit portfolio losses. It has also served as the basis for the capital charge formulae or "capital curves" contained in the Basel II proposals. Any bank regulated under Basel II will calculate the regulatory capital it must hold against a particular credit exposure by plugging variables describing the exposure (such as default probability and loss given default) into the Vasicek formula.

A drawback with the simple Vasicek model is that it is a purely static, one-period model. In practice, portfolio default rates move in a predictable way from period to period, and exhibit quite well-defined time series properties. Ignoring these properties in risk and capital calculations may lead to an erroneous perception of true risk as one may fail to identify which fraction of volatility in losses is a true innovation and which is forecastable given current information.

The contributions of this paper are:

1. To generalise the Vasicek model to allow for autocorrelated loan losses. Formulating models in which the underlying factors driving the latent variables are

autoregressive processes, we show that the transformed loss rates inherit this time series structure.

2. To implement the model empirically using loan loss data for US banks. We apply a Maximum Likelihood (ML) approach to estimate the parameters of aggregate portfolio loss rate distributions. ML is equivalent, in this case, to a simple OLS regression of the transformed loss on its own lags.

3. To study the correlation structure of innovations to losses for the six different loan categories in our sample. This sheds light on the degree to which credit risk may be diversified across different credit sectors.

4. To implement a conditional version of the extended, dynamic Vasicek model. This model implies that transformed loss rates are linear regressions of observable, cyclical factors such as shocks to IP and unemployment.

5. To show how the conditional version of the model may be used to devise stress scenarios for loan portfolios.

6. To investigate the implications of our approach for the measurement of risk in loan portfolios and the calculation of adequate capital. In this, we compare the implied capital with the regulatory capital charges implied by the Basel II proposals.

One may compare our analysis with previous empirical studies of bond and loan defaults. Broadly speaking, the different studies may be categorized into latent variable or hazard rate approaches. In latent variable models, defaults occur when a variable, more or less closely identified with the borrower's underlying asset value, crosses a threshold. By contrast, in a hazard rate model, defaults occur when a point process, in most cases conditionally Poisson, jumps.

Latent variable models have been used to investigate individual default behaviour have included Altman (1968), Ohlson (1980), Kealhofer (2003), Hillegeist, Keating, Cram, and Lundstedt (2004) and Bartram, Brown, and Hund (2007). Such models have also been employed specifically to study correlations between individual defaults (for example, Zhou (2001) and de Servigny and Renault (2002)).

Several studies have employed latent variable models to examine how default probabilities and ratings transitions are related to business cycles. Nickell, Perraudin, and Varotto (2000), Koopman and Lucas (2005) and Koopman, Lucas, and

Klaassen (2005) study how transition probabilities and default rates are linked to macro-economic drivers. Pesaran, Schuermann, Treutler, and Weiner (2006) and Pesaran, Schuermann, and Weiner (2004) develop a multi-country macro model and link its output to default data and bank loan portfolio performance. This enables them to make statements about how defaults behave conditional on particular stress scenarios.

Recent latent variable studies, close in some respects to ours, include McNeil and Wendin (2006) and McNeil and Wendin (2007). These papers investigate corporate defaults and ratings transitions using latent-variable, logit models in which the underlying factors are serially correlated. Estimating these models on default and transition data for individual firms (rather than on aggregate default or transition rates) is challenging and the authors develop Bayesian methods using the Gibbs sampler and then relate the results to the evolution of the business cycle. In McNeil and Wendin (2007), they argue that there is evidence of a latent cyclical variable affecting defaults over and above the influence of a proxy for the macroeconomic cycle that they devise.

The application of hazard-based models to the empirical investigation of corporate defaults is a very active area of recent research. Early studies include Lane, Looney, and Wansley (1986) who apply a Cox proportional hazard model in studying bank failures. Lando and Skodeberg (2002) models corporate ratings transitions using a continuous-time, hazard-based approach. Couderc and Renault (2004) study times to default and default probabilities and examine, amongst other aspects, how different variables have impacts over different time spans. Lee and Urrutia (1996) compare hazard and logit approaches in modelling corporate defaults.

Important recent contributions to the hazard rate literature include Duffie, Saita, and Wang (2007) who investigate empirically simple hazard-based models in which the only source of correlation between firms is observable variables. They apply this data to default probabilities extracted from an equity-based model and conclude that the degree of correlation between default probabilities can only be explained if there are unobserved factors that they term frailty variables.

Duffie, Eckner, Horel, and Saita (2006) estimate the term structure of corporate default probabilities based on a range of firm-specific and market wide factors within a hazard-based framework. Das, Duffie, Kapadia, and Saita (2007) investigate factor default risk in US corporate bonds and attribute a large fraction of co-movements in default probabilities to unobserved frailty variables.

Finally, note that some studies refer to discrete-choice modelling (for example, logit or probit) as a 'hazard-based approach'. In our terminology, such models are 'latent variable' models in that defaults occur when latent variables cross thresholds. For example, Schumway (2001) uses a discrete time logit model to investigate corporate defaults combining accounting and market explanatory variables. Chava and Jarrow (2004) use industry and monthly (as opposed to annual data) in discrete choice models like that of Schumway (2001).

Recently, there has been growing interest in understanding how bank loans like those we study in this paper differ from other types of defaultable debt. Carey (1998) compares default rates and loss severity for US bank loans and bonds, focussing on the relative riskiness of public versus private debt. Ruckes (2004) examines cyclical patterns in bank lending standards, tracing the evolution of standards to the incentives banks face and cyclical evolution in the profitability of screening. Becker (2007) studies regional segmentation in US bank loan markets, examining the effects of local deposit supply on loan availability. While Gross and Souleles (2002) investigates time to default in a US credit card book using duration modeling.

The structure of this paper is as follows. Section 2 derives the dynamic default rate distributions. Section 3 describes empirical implementation of these distributions on aggregate US bank loan losses. Section 4 analyses the factors implicit in the loan loss series, examining correlation across different loan market sectors, for example. Section 5 implements a conditional version of the dynamic loan loss distribution, regressing transformed default rates on macroeconomic variables and studying how default rate distributions may be stressed. Section 6 looks at the implications of our analysis for bank capital.

2 Loan Loss Models

2.1 Loan Losses with Autocorrelation and Trends

In this section, we generalise the arguments of Vasicek (1991) to allow for autocorrelated common factors. The pattern of autocorrelation is then inherited by aggregate loss rates, appropriately transformed. Vasicek's model has been extended in other dimensions by Koopman, Lucas, and Klaassen (2005) who deduce distributions of aggregate ratings transitions and by Schonbucher (2002) who considered default rates when underlying risk factors are non-Gaussian.

Suppose there are n obligors. If not in default at $t-1$, the ith obligor defaults at t if a latent variable, $Z_{i,t}$ satisfies $Z_{i,t} < c$ for a constant c. Suppose that the $Z_{i,t}$ for $t = 0, 1, 2, \ldots$ and $i = 1, 2, \ldots, n$ satisfy a factor structure in that:

$$Z_{i,t} \;=\; \sqrt{\rho} X_t \;+\; \sqrt{1-\rho}\epsilon_{i,t} \;. \tag{1}$$

Assume that:

$$X_t \;=\; \sqrt{\beta} X_{t-1} \;+\; \sqrt{\lambda}\eta_t \tag{2}$$

where $\epsilon_{i,t}$ and η_t are standard normal and independent for pairs of obligors i and j and for all dates t.

Typically, when latent variable models such as that described by equation (1) are formulated, the shocks are assumed to have unit variance. We construct our model so that X_t has unit unconditional variance. Since

$$X_t \;=\; \sum_{i=0}^{\infty} \left(\sqrt{\beta}\right)^i \sqrt{\lambda}\eta_{t-i} \tag{3}$$

$$\text{Variance}(X_t) \;=\; \sum_{i=0}^{\infty} \beta^i \lambda \;=\; \frac{\lambda}{1-\beta} \tag{4}$$

if we choose $\lambda = 1 - \beta$, the unconditional variance of X_t is unity and

$$X_t \;=\; \sqrt{\beta} X_{t-1} \;+\; \sqrt{1-\beta}\eta_t \;. \tag{5}$$

The notion that underlying factors may be autoregressive is similar to the approach of McNeil and Wendin (2006) and McNeil and Wendin (2007). If X_t has unit variance unconditionally, the $Z_{i,t}$ has a standard normal unconditional distribution. The unconditional probability of default for the ith obligor, q, then satisfies:

$$\Phi^{-1}(q) = c. \tag{6}$$

Now, consider the probability of default for i in our model conditional on information at $t-1$. Default occurs when:

$$\sqrt{\rho} X_t \;+\; \sqrt{1-\rho}\epsilon_{i,t} \;<\; c \tag{7}$$

$$\sqrt{\rho}\sqrt{1-\beta}\eta_t \;+\; \sqrt{1-\rho}\epsilon_{i,t} \;<\; c \;-\; \sqrt{\rho}\sqrt{\beta} X_{t-1} \;. \tag{8}$$

The left hand side of equation (8) is distributed as $\mathrm{N}(0, 1-\rho\beta)$. So the default probability conditional on information at $t-1$ is:

$$q_{i,t} \;=\; \Phi\left(\frac{c-\sqrt{\rho}\sqrt{\beta}X_{t-1}}{\sqrt{1-\rho\beta}}\right) . \tag{9}$$

Conditional on the common factor η_t and on X_{t-1}, defaults are independent across individual obligors. So, denoting $P(k,n)$ as the probability of observing k defaults out of n obligors conditional on X_{t-1} and removing the conditioning of the shock, η_t, by integrating over its support, gives:

$$\begin{aligned} P(k,n) \;=\; & \binom{n}{k} \int_{-\infty}^{\infty} \Phi\left(\frac{c-\sqrt{\rho}\sqrt{\beta}X_{t-1}-\sqrt{\rho}\sqrt{1-\beta}\eta_t}{\sqrt{1-\rho}}\right)^k \\ & \times \left[1-\Phi\left(\frac{c-\sqrt{\rho}\sqrt{\beta}X_{t-1}-\sqrt{\rho}\sqrt{1-\beta}\eta_t}{\sqrt{1-\rho}}\right)\right]^{n-k} d\Phi(\eta_t) . \end{aligned} \tag{10}$$

Adopting the change of variables:

$$s(\eta) \;\equiv\; \Phi\left(\frac{c-\sqrt{\rho}\sqrt{\beta}X_{t-1}-\sqrt{\rho}\sqrt{1-\beta}\eta}{\sqrt{1-\rho}}\right) . \tag{11}$$

Then:

$$\begin{aligned} P(k,n) \;=\; & -\binom{n}{k} \int_0^1 s^k (1-s)^{n-k} \\ & \times\; d\Phi\left(\frac{-\left(\sqrt{1-\rho}\Phi^{-1}(s)-c+\sqrt{\rho}\sqrt{\beta}X_{t-1}\right)}{\sqrt{\rho}\sqrt{1-\beta}}\right) . \end{aligned} \tag{12}$$

But:

$$-\,d\Phi(f(s)) \;=\; d\Phi(-f(s)) \tag{13}$$

so

$$P(k,n) \;=\; \binom{n}{k} \int_0^1 s^k (1-s)^{n-k}\, dW(s) , \tag{14}$$

where

$$W(s) \equiv \Phi\left(\frac{\sqrt{1-\rho}\Phi^{-1}(s)-c+\sqrt{\rho}\sqrt{\beta}X_{t-1}}{\sqrt{\rho}\sqrt{1-\beta}}\right) \tag{15}$$

Now, consider what happens as $n \to \infty$. Let θ denote the fraction of the pool that defaults. Then:

$$\lim_{n\to\infty} \sum_{i=0}^{[n\theta]} P(i,n) \;=\; \int_0^1 \left(\lim_{n\to\infty} \sum_{i=0}^{[n\theta]} \binom{n}{i} s^i (1-s)^{n-i}\right) dW(s) \tag{16}$$

$$= \int_0^1 1(s < \theta) dW(s) \tag{17}$$

$$= W(\theta) - W(0) \;\; = \;\; W(\theta) \; . \tag{18}$$

Hence, the loss distribution conditional on X_{t-1} is:

$$W(\theta_t) \equiv \Phi\left(\frac{\sqrt{1-\rho}\Phi^{-1}(\theta_t) - \Phi^{-1}(q) + \sqrt{\rho}\sqrt{\beta}X_{t-1}}{\sqrt{\rho}\sqrt{1-\beta}}\right) \; . \tag{19}$$

So the transformed loss rate $\tilde{\theta}_t \equiv \Phi^{-1}(\theta_t)$ is Gaussian and satisfies:

$$\tilde{\theta}_t \equiv \Phi^{-1}(\theta_t) \sim \mathrm{N}\left(\frac{\Phi^{-1}(q) - \sqrt{\rho}\sqrt{\beta}X_{t-1}}{\sqrt{1-\rho}} \; , \; \frac{\rho(1-\beta)}{1-\rho}\right) \; . \tag{20}$$

One may express this as:

$$\tilde{\theta}_t \;\; = \;\; \frac{\Phi^{-1}(q) - \sqrt{\rho}\sqrt{\beta}X_{t-1}}{\sqrt{1-\rho}} \;\; - \;\; \frac{\sqrt{\rho}\sqrt{1-\beta}}{\sqrt{1-\rho}}\eta_t \tag{21}$$

where η_t is the shock in equation (2) above. Note that the negative sign before the last term enters because of our change of variable $-d\Phi(\eta(s)) = d\Phi(-\eta(s))$.

Solving (21) for X_{t-1}, we obtain:

$$X_{t-1} \;\; = \;\; \frac{1}{\sqrt{\rho}\sqrt{\beta}}\left[\Phi^{-1}(q) - \sqrt{1-\rho}\tilde{\theta}_t - \sqrt{\rho}\sqrt{1-\beta}\eta_t\right] \; . \tag{22}$$

Lagging this equation and substituting in

$$X_{t-1} \;\; = \;\; \sqrt{\beta}X_{t-2} + \sqrt{1-\beta}\eta_{t-1} \tag{23}$$

yields the following result:

Proposition 1 *Suppose that an individual obligor who is not in default at* $t-1$ *defaults at* t *if a latent variable* $Z_{i,t} < c$. *Assume that the* $Z_{i,t}$ *satisfy the Gaussian autoregressive factor structure given by equations (1) and (2). Let* θ_t *denote the loss rate, i.e., the fraction of obligors that default. As* $n \to \infty$, *the transformed loss rate,* $\tilde{\theta}_t = \Phi^{-1}(\theta_t)$ *converges to the following Gaussian order-1 autoregressive process:*

$$\tilde{\theta}_t = \sqrt{\beta}\tilde{\theta}_{t-1} + \frac{(1-\sqrt{\beta})}{\sqrt{1-\rho}}\Phi^{-1}(q) \;\; - \;\; \frac{\sqrt{\rho}\sqrt{1-\beta}}{\sqrt{1-\rho}}\eta_t \; . \tag{24}$$

Hence, the transformed loss rate at t conforms to the following Gaussian distribution:

$$\tilde{\theta}_t = \Phi^{-1}(\theta_t) \sim \mathrm{N}\left(\sqrt{\beta}\tilde{\theta}_{t-1} + \frac{(1-\sqrt{\beta})}{\sqrt{1-\rho}}\Phi^{-1}(q)\ ,\ \frac{\rho(1-\beta)}{1-\rho}\right)\ . \tag{25}$$

Now, suppose that the cut-off point for default evolves deterministically over time in that an obligor not in default at $t-1$ defaults at t if $Z_{i,t} < c(t)$ for a function $c(.)$. Define $q_t \equiv \Phi(c(t))$. Then, one may easily show that $\tilde{\theta}_t$ follows the process:

$$\tilde{\theta}_t = \sqrt{\beta}\tilde{\theta}_{t-1} + \frac{\Phi^{-1}(q_t)}{\sqrt{1-\rho}} - \sqrt{\beta}\frac{\Phi^{-1}(q_{t-1})}{\sqrt{1-\rho}} - \frac{\sqrt{\rho}\sqrt{1-\beta}}{\sqrt{1-\rho}}\eta_t\ . \tag{26}$$

This process is autoregressive around a non-linear trend.

2.2 The General AR Case

The above analysis may be generalized to allow for any autoregressive process for the factor, X_t. Suppose that:

$$X_t \ = \ \sqrt{\alpha}B(L)X_{t-1} \ + \ \sqrt{1-\alpha}\eta_t \tag{27}$$

where

$$B(L)\ X_{t-1} \ \equiv \ \sum_{i=1}^{M}\xi_i X_{t-i}\ . \tag{28}$$

Choose $\sqrt{\alpha}$, ξ_i $i = 1, 2, \ldots, M$ so that the unconditional variance of X_t is unity.

Similar arguments to those used before imply:

$$\tilde{\theta}_t \ = \ \frac{\Phi^{-1}(q_t) - \sqrt{\rho}\sqrt{\alpha}B(L)X_{t-1}}{\sqrt{1-\rho}} - \frac{\sqrt{\rho}\sqrt{1-\alpha}}{\sqrt{1-\rho}}\eta_t\ . \tag{29}$$

If the roots of the polynomial lag operator $B(L)$ lie outside the unit circle (see Hamilton (1994)), then:

$$X_{t-1} \ = \ B^{-1}(L)\left(\frac{1}{\sqrt{\rho}\sqrt{\alpha}}\left[\Phi^{-1}(q_t) - \sqrt{1-\rho}\tilde{\theta}_t - \sqrt{\rho}\sqrt{1-\alpha}\eta_t\right]\right) \tag{30}$$

Lagging this equation by one period, substituting for X_{t-1} and X_{t-2} in:

$$X_{t-1} \ = \ \sqrt{\alpha}B(L)X_{t-2} \ + \ \sqrt{1-\alpha}\eta_{t-1}\ , \tag{31}$$

and then rearranging, one obtains:

Proposition 2 *Under the above assumptions, when X_t follows the general autoregressive process in equation (27), the transformed loss rate, $\tilde{\theta}_t \equiv \Phi^{-1}(\theta_t)$, converges to the process*

$$\tilde{\theta}_t = \sqrt{\alpha}B(L)\tilde{\theta}_{t-1} + \frac{\Phi^{-1}(q_t)}{\sqrt{1-\rho}} - \sqrt{\alpha}B(L)\frac{\Phi^{-1}(q_{t-1})}{\sqrt{1-\rho}} \; - \; \frac{\sqrt{\rho}\sqrt{1-\alpha}}{\sqrt{1-\rho}}\eta_t \; . \tag{32}$$

as the number of loans $n \to \infty$.

3 Empirical Implementation

3.1 Data and Estimation

To implement our model, we employ quarterly aggregate charge-off data for all US banks for the period 1985 to 2007. This is published by the Federal Reserve Board. Note, that the data is presented net of recoveries. We therefore scale each of the loss series by dividing through by the Loss Given Default (LGD) for each series. The assumed LGDs were taken from the Basel Committee on Banking Supervision (1999) and can be found in Table 9.

There may be problems with using charge-off rates. When a new manager takes over a division of a bank, he or she may wish to write off delinquent and semi-delinquent loans in order to be able to demonstrate a better performance subsequently. The fact that we employ data that aggregates charge-offs from many banks will mitigate this problem.

Plots of the six charge-off rate series are shown in Figure 1. In general, visual inspection of the series seems to confirm the presence of trends and cyclical behavior in loss rates, reinforcing the basic point of this paper that the unconditional and conditional distributions of losses are very different.

To estimate the simple static Vasicek loss distribution, one may use the fact that under the Vasicek assumptions, the transformed loss rate, $\tilde{\theta}_t$ is normally distributed with a standard deviation that depends in a simple way on the asset correlation. So regressing $\tilde{\theta}_t \equiv \Phi^{-1}(\theta_t)$ on a constant, one may infer the unconditional default probability from the coefficient on the constant and the correlation parameter ρ from the standard error of the regression residuals.

It is easy to extend this to cases in which the transformed loss rate is autoregressive

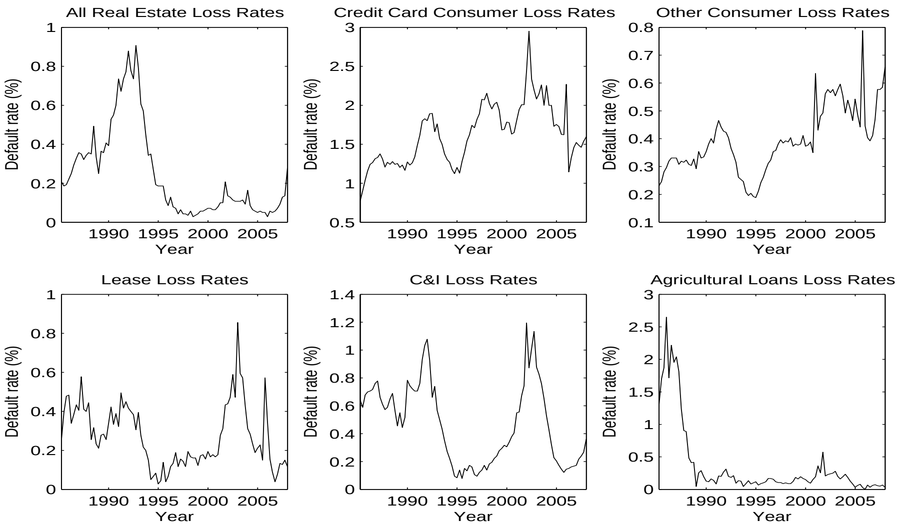

Figure 1: **Calibration Loan Loss Series**

Plots of aggregate US banks quarterly loan loss rates from 1985 to fourth quarter 2007. Two models were calibrated using this data, one without autocorrelation, so a static model, and then another model assuming autocorrelation in the losses.

simply by regressing on lagged $\tilde{\theta}_t$ as well as on a constant. Equation (25) shows the Gaussian distribution that the transformed loss rate follows for the dynamic distribution case. Here the transformed loss rate is conditionally Gaussian, and, in the AR(1) case, one may infer the unconditional default probability q, the square of the factor loading on its lagged level, β, and the correlation parameter ρ from the regression constant, the regression coefficient on $\tilde{\theta}_{t-1}$ and the standard error of the regression residuals.

3.2 Results

The estimation results for the various quarterly series are contained in Table 1. The first two rows show the unconditional mean and the unconditional standard deviations of loss rates. Credit Cards, (CC), exhibit a much lower credit quality than the other loan categories in that the mean loss is 1.63%. By contrast, Agricultural loans, (A), for example, exhibit a loss rate of just 0.36%. But the volatility of the Credit Card loss rate (at 0.39%) is of a similar order of magnitude to the other categories. This might suggest that Credit Card loans are riskier that other loan types. But, as we see below, risk as measured for example by Value at Risk and capital depends in a complex way

Table 1: **Parameter Estimations**

Parameter estimates based on various aggregate quarterly US bank loan loss series. Estimates were performed using both models with and without autocorrelation in the losses.

	RE	CC	OC	L	CI	A
Series Statistics						
Loss rate series mean (%)	0.25	1.63	0.39	0.27	0.47	0.36
Loss rate std dev (%)	0.23	0.39	0.12	0.16	0.29	0.56
Parameter Estimates with No Autocorrelation						
Regression residual volatility (%)	30.12	9.62	10.17	21.74	24.39	40.37
Factor correlation ρ (%)	8.32	0.92	1.02	4.51	5.62	14.01
Std error ρ (%)	1.13	0.13	0.15	0.64	0.79	1.79
Unconditional default probability q (%)	0.25	1.63	0.39	0.28	0.48	0.35
Std error q (%)	0.07	0.09	0.03	0.05	0.09	0.12
Parameter Estimates with Autocorrelation						
Regression residual volatility (%)	8.27	4.25	4.61	12.19	6.95	25.77
Factor correlation ρ (%)	8.67	0.64	1.05	4.66	5.30	13.49
Std error ρ (%)	6.22	0.20	0.48	1.59	3.41	3.30
Unconditional default probability q (%)	0.27	1.69	0.44	0.27	0.41	0.30
Std error q (%)	0.21	0.13	0.07	0.06	0.20	0.10
AR(1) parameter β (%)	92.80	71.84	79.97	69.62	91.37	57.40
Std error β (%)	5.55	7.85	8.67	9.86	5.71	10.22

on default probability and default correlation, ρ, and cannot be summarized simply by volatility.

Transforming the loss rate series by applying $\Phi^{-1}(.)$ (the inverse of the standard normal cdf) reveals that the Credit Card transformed loss rate volatility, which is the standard deviation of the estimating regression residuals, is less than those for other series. This then translates into a smaller correlation parameter for Credit Cards, 0.92%, compared to 14.01% for Agricultural and 8.32% for Real Estate, (RE). Other Consumer loans, (OC), has a correlation parameter of 1.02% while Corporate and Industrial loans, (CL), and Leases, (L), (that are both primarily corporate) have correlation parameters of 5.62% and 4.51%.

In the lower part of Table 1, results are reported for models that include autocorrelation. Introducing autocorrelation substantially reduces the standard errors of the transformed loss rates residuals. For example, it falls from 30.12% to 8.27% in the case of the Real Estate loans. This is to be expected as conditioning on an additional regressor, the lagged loss, the residual volatility of the series should fall. Perhaps surprisingly since they reflect the variability in loss rates, the correlation parameters ρ are broadly comparable to those from the model without autocorrelation. This does not mean, however, that the risk characteristics of the loss rates is the same as the series are now mean-reverting.

4 Factors and Correlations

4.1 Specification Tests

From each transformed loss rate time series, one may use the equation

$$\tilde{\theta}_t = \frac{\Phi^{-1}(q) - \sqrt{\rho} X_t}{\sqrt{1-\rho}} . \tag{33}$$

to infer the underlying factor value in each period. We calculate the factors driving both the static and autocorrelation models for each loan category. In the autocorrelation case it is actually the common factor innovations that we deduce:[1]

$$\eta_t = \frac{X_t - \sqrt{\beta} X_{t-1}}{\sqrt{1-\beta}} . \tag{34}$$

Figure 2 shows the factor innovations over time. If the autocorrelation model is correct then the first order lag should remove all time dependencies from the data and the shocks should follow a white noise process. Inspection suggests that there may be some clustering of shocks in Leases and Agricultural loans and possibly some higher order autocorrelation in Credit Card receivables.

To test the specification of the transformed loan loss process in equation (24), we calculated Durbin-Watson test statistics for the regression residuals. The Durbin-Watson statistic tests for autocorrelation in the residuals of a regression. Table 2 shows the test statistics for all the loan loss series. In each case, one may reject the null that autocorrelation is present in the residuals at a 95% confidence limit, which suggests that the factor innovations are indeed independent.

As additional specification tests, we perform normality tests on the innovations from the different models (see Table 2) in that we implement Jarque-Bera test. This

[1]An initial value of $X_0 = 0$ was assumed for each factor series.

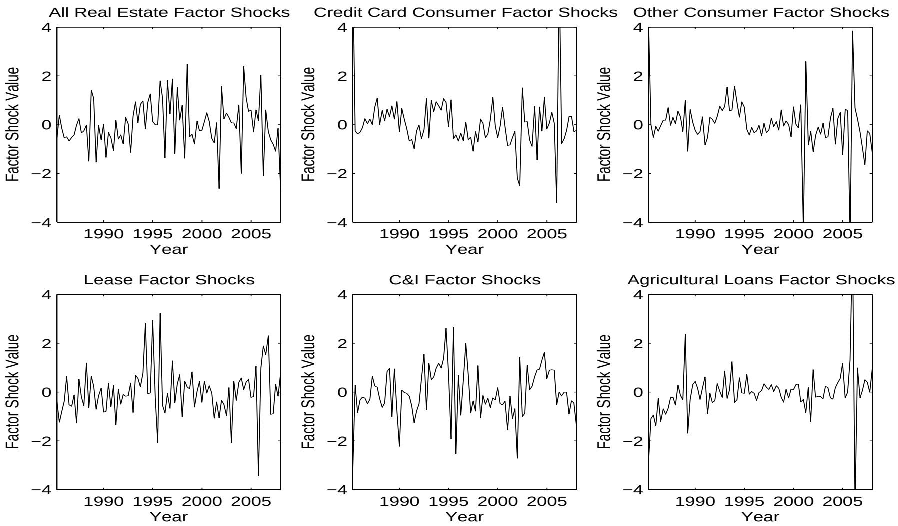

Figure 2: **Autoregressive Factor Shocks**

Plots of the common factor autoregressive innovations extracted from the US banks aggregate quarterly loan loss rates. The inferred shocks are from 1985 to fourth quarter 2007.

assesses whether the sample skewness and kurtosis differ to a statistically significant degree from their values under normality. We generate confidence levels for the Jarque-Bera test using Monte Carlos rather than using the asymptotically valid confidence level. The Monte Carlo confidence level for rejection of normality is 5.99.

The Durbin-Watson and the Jarque-Bera tests suggests no rejections of the null hypothesis that the residuals are uncorrelated and Gaussian.

4.2 Factor Correlations

It is interesting to examine how correlated are the factors driving the different categories of loan losses. This can shed light on the degree to which diversifying across different sectors of the loan market will help a bank reduce its risk. In asking such questions, our analysis is comparable to that of Rosenberg and Schuermann (2006) who consider the degree of correlation across broad categories of banking risk such as operational, market and credit risk.

Table 3 reports correlation matrices of (i) the static-model factor series and (ii) factor innovations from the dynamic model, in both cases based on quarterly data. The correlation matrix for the static model without autocorrelation appears plausible.

Table 2: **Hypothesis Testing on the Autoregressive Shocks**

Various hypothesis tests performed on the quarterly US bank loss rates. The Durbin-Watson test statistic permits one to assess the null hypothesis that shocks are uncorrelated. A statistic less than the confidence level of 1.679 implies the null is not rejected at 95%. The Jarques-Bera test assesses whether higher moments differ from normal distributions values. The confidence limit for rejection is 5.99 at 95%. *.

	RE	CC	OC	L	CI	A
Durbin-Watson	2.29	2.34	2.57	2.23	1.89	2.54
Jarques-Bera	5.23	4.94	4.66	5.00	4.91	5.13

The pattern of correlations is reasonable in that Credit Card losses are most highly correlated with Other Consumer Loans. All Real Estate is fairly highly correlated with most categories. The corporate loan categories, Leases, C&I and Agricultural are reasonably highly correlated.

The magnitudes of the correlations are generally lower in the dynamic model, except for the correlations of Agriculture Loans with Real Estate and Credit Card Loans. This reflects the fact that the shocks in the static model may be thought of as weighted sums of true innovations. These are more correlated across sectors than the innovations themselves, underlining our point that that unconditional and conditional risks are very different.

To analyze the pattern of correlation further, we decompose the quarterly correlation matrix, in the two cases with and without autocorrelation, into their principal components (see Table 4). To analyze how far the correlation matrix is from one with a single or two factors up to many, one may examine the magnitudes of the largest eigenvalues. One may also see which of the loan loss categories seem to be driven by the different factors.

First we performed a hypothesis test to assess the magnitudes of the correlation matrix eigenvalues. This sheds light on how many economically significant common factors are present in the factors driving the individual loan loss rates. More details of the test statistic may be found in Appendix A. In brief, the statistic tests whether, at particular confidence levels, the principal components associated with different eigenvalues contributes significant amounts of common variability.

First the eigenvalues are arranged in ascending order. The smallest p eigenvalues are summed and their total is tested against the hypothesis that the total is negligible.

Table 3: **Loss Rate Correlation Matrices**
Correlation matrices of the common factors extracted from models with and without autocorrelation using quarterly US bank loss rates. In the static case the correlation between the common factors are calculated. In the autocorrelated case its the correlation between the factor shocks

	RE	CC	OC	L	CI	A
Correlation Matrix with No Autocorrelation						
RE	1.00	-0.40	-0.33	0.37	0.61	0.37
CC	-0.40	1.00	0.66	0.20	0.06	-0.33
OC	-0.33	0.66	1.00	0.38	0.22	-0.23
L	0.37	0.20	0.38	1.00	0.79	0.41
CI	0.61	0.06	0.22	0.79	1.00	0.55
A	0.37	-0.33	-0.23	0.41	0.55	1.00
Correlation Matrix with Autocorrelation						
RE	1.00	-0.18	0.11	0.16	0.28	0.40
CC	-0.18	1.00	0.26	0.10	-0.10	-0.57
OC	0.11	0.26	1.00	0.29	0.06	0.00
L	0.16	0.10	0.29	1.00	-0.07	0.04
CI	0.28	-0.10	0.06	-0.07	1.00	0.11
A	0.40	-0.57	0.00	0.04	0.11	1.00

Table 5 shows the number of eigenvalues that reject the hypothesis at various confidence levels. The tests suggest that in both the static and dynamic model, multiple principal components play a significant role in the correlation between the loan loss series.

Examining Table 4, one may note that the retail type categories, Credit Cards and Other Consumer have the same sign loadings on the factors that contribute the most correlation, negative for the most important factor and positive for the next most important. The corporate loan categories, C&I and Leases mainly have positive weights on these factors too (except for Leases in the dynamic case that has a small negative). The largest factor could be deemed as differentiating across business sectors whilst the next could be more of a business cycle effect.

Table 4: **Principal Component Analysis of Factor Correlations**
Principal component analysis of factor correlation matrix inferred from quarterly US loss data using both a model with and without autocorrelation in the factor. Each row is an unobserved factor that is common to all loss series and each element is then the loading or weight of that factor for a given series.

	Eigenvalue	RE	CC	OC	L	CI	A
Eigenvalue and Factor Weights with No Autocorrelation							
Common Factor 1	0.11	1.42	0.43	0.77	0.57	-2.25	0.90
Common Factor 2	0.23	0.54	0.03	1.01	-1.59	0.51	0.55
Common Factor 3	0.34	-0.25	-1.33	0.94	0.34	-0.13	-0.30
Common Factor 4	0.64	0.80	0.04	-0.01	-0.02	0.13	-0.95
Common Factor 5	2.08	-0.16	0.41	0.44	0.24	0.15	-0.11
Common Factor 6	2.60	0.29	-0.09	-0.02	0.30	0.35	0.28
Eigenvalue and Factor Weights with Autocorrelation							
Common Factor 1	0.41	0.61	-0.76	0.33	-0.14	-0.26	-1.15
Common Factor 2	0.66	0.74	0.46	-0.02	-0.64	-0.53	0.28
Common Factor 3	0.75	0.49	0.22	-0.78	0.64	0.03	-0.20
Common Factor 4	1.13	-0.15	-0.20	0.12	0.42	-0.77	0.21
Common Factor 5	1.61	0.34	0.14	0.53	0.37	0.21	0.15
Common Factor 6	2.30	0.21	-0.47	-0.13	-0.04	0.13	0.36

5 Conditional Default Rate Distributions

5.1 Conditioning on Macroeconomic Variables

Risk analysis of a bank's portfolio is usually performed by calculating statistics such as VaRs or Expected Shortfalls for the distribution of the future portfolio value. These statistics are typically calculated on an unconditional basis, i.e., without trying to conditional on possible future events.

It is also interesting, however, to consider conditional risk statistics. One may wish to evaluate how the portfolio will behave in the event of a recession (as measured by a given deterioration in macroeconomic variables like unemployment or output growth). Such analysis is often referred to as stress testing. Research on stress testing methodologies includes Kupiec (1998), Berkowitz (1999), Longin (2000) and Peura and Jokivuolle (2004). Among recent empirical papers on default probabilities, Pesaran, Schuermann, Treutler, and Weiner (2006) and Pesaran, Schuermann, and Weiner (2004) consider analysis of risk in loan and bond portfolios conditional on

Table 5: **Hypothesis Testing of Eigenvalues**

Hypothesis testing to reject the negligibility of the smaller principal components of the factor correlation matrix. Appendix A describes the testing hypothesis. Values at each of the confidence limits are the number of eigenvalues that reject the hypothesis.

Confidence Limit	75%	85%	90%	95%
Model with No Autocorrelation	4	3	3	2
Model with Autocorrelation	3	2	1	1

macroeconomic shocks and hence, in a sense, devise stress test methods for credit portfolios.

The approach we develop in this section of allowing default rates to be driven by a combination of observable macroeconomic factors and unobservable, common factors is related to recent research, summarised in the introduction using the hazard rate approach to investigate bond defaults and default probabilities. In particular, Duffie, Saita, and Wang (2007) suppose that fluctuation in observable variables induce correlations between default probabilities. Das, Duffie, Kapadia, and Saita (2007) investigate whether defaults generated by hazards with observable driving variables are more correlated than one might expect. Duffie, Eckner, Horel, and Saita (2006) show how one may estimate hazard models with a combination of observed and unobserved driving processes.

In this section, we consider how our loan loss distributions may be expressed in conditional terms suitable for framing and analysing stress scenarios. Recall that obligor i is assumed to default at time t based on equation (1). Now, suppose that the latent random variable, $Z_{i,t}$, is made up of the sum of observed and unobserved factors:

$$Z_{it} = \sqrt{\rho}\left(\sqrt{1-\lambda^2}X_t + \lambda\sum_{j=1}^{J} a_j^* Y_{j,t}\right) + \sqrt{1-\rho}\epsilon_{i,t} \ . \qquad (35)$$

We suppose that the $Y_{j,t}$ are observable macroeconomic variables. We shall condition on these variables so it is not necessary to specify exactly what processes these variables follow. $X_{i,t}$ is a first-order autoregressive stochastic process as before and $\epsilon_{i,t}$ and η_t are assumed to be independent standard normals. λ determines the contribution of the observed and unobserved factors to $Z_{i,t}$. It is assumed that $\lambda \in (0,1)$.

To ensure that unconditionally, the $Z_{i,t}$ have unit variance, we re-scale the $\sum_{j=1}^{J} a_j^* Y_{j,t}$

term by redefining the observable factor weights as:

$$a_j \equiv \frac{a_j^*}{\sqrt{\sum_{k=1}^{J}\sum_{m=1}^{J} a_k a_m \text{Covariance}(Y_{k,t}, Y_{m,t})}} \tag{36}$$

and modifying the default driver of obligor i to:

$$Z_{it} = \sqrt{\rho}\left(\sqrt{1-\lambda^2}X_t + \lambda a'Y\right) + \sqrt{1-\rho}\epsilon_{i,t}\ . \tag{37}$$

Here, the summation, $\sum a_j Y_j$, has been suppressed in that the factors and their weights are expressed as vectors. The $Y_{j,t}$ are assumed to be stationary series so the covariance matrix is independent of time.

The derivation of the dynamic process for the loss rate conditional on the macroeconomic factors closely follows that of Section 2. The loss rate distribution conditional on X_{t-1} and Y may be shown to equal:

$$W(\theta_t) \equiv \Phi\left(\frac{\sqrt{1-\rho}\Phi^{-1}(\theta_t) - \Phi^{-1}(q) + \sqrt{\rho}\sqrt{1-\lambda^2}\sqrt{\beta}X_{t-1} + \sqrt{\rho}\lambda aY}{\sqrt{\rho}\sqrt{1-\lambda^2}\sqrt{1-\beta}}\right)\ . \tag{38}$$

The transformed loss rate $\tilde{\theta}_t \equiv \Phi^{-1}(\theta_t)$, therefore, conforms to the following Gaussian distribution:

$$\text{N}\left(\frac{\Phi^{-1}(q) - \sqrt{\rho}\sqrt{1-\lambda^2}\sqrt{\beta}X_{t-1} - \sqrt{\rho}\lambda aY}{\sqrt{1-\rho}},\ \frac{\rho(1-\lambda^2)(1-\beta)}{1-\rho}\right)\ . \tag{39}$$

From this, the transformed loss rate may be expressed in terms of its mean, standard deviation and a shock. This can then be rearranged, lagged and substituted into equation (2) to derive an autoregressive process for the transformed loss rate conditional upon observable stochastic processes Y_t:

Proposition 3 *When individual-borrower latent variables, Z_{it}, and the unobserved, common factor X_t, follow the processes given by equations (35)and (5), respectively, as the number of loans $n \to \infty$, the transformed loss rate, $\tilde{\theta}_t \equiv \Phi^{-1}(\theta_t)$, converges to the process*

$$\begin{aligned}\tilde{\theta}_t &= \sqrt{\beta}\tilde{\theta}_{t-1} + \frac{1-\sqrt{\beta}}{\sqrt{1-\rho}}\Phi^{-1}(q) \\ &\quad - \frac{\sqrt{\rho}}{\sqrt{1-\rho}}\lambda a(Y_t - \sqrt{\beta}Y_{t-1}) - \frac{\sqrt{\rho}\sqrt{1-\lambda^2}}{\sqrt{1-\rho}}\sqrt{1-\beta}\eta_t\ . \end{aligned} \tag{40}$$

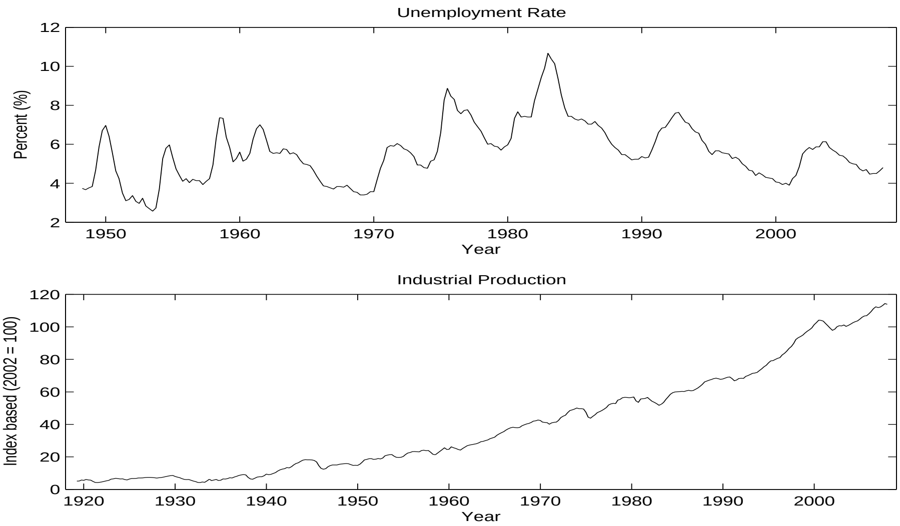

Figure 3: **Macro-Economic Stress Parameters**
Macro-economic cyclical indicators used in the model to stress on observable factors. The top panel is the US Unemployment Rate over the period 1948 to 2007 and the lower panel is the US Industrial Production over the period 1919 to 2007 as an index, 2002 = 100.

5.2 Estimation

We implement the model described above using two of the loss rate series previously described: Credit Card losses and C&I losses. As before, the series were re-scaled by the LGD estimates contained in Basel Committee on Banking Supervision (1999).

Though it is straightforward in our framework to create a multi-factor stress, conditioning on several observed macroeconomic variables, for clarity of the results, we restrict ourselves to a univariate and a bivariate case. Specifically, we consider stresses involving the US Employment Rate and US Industrial Production.[2] The series come respectively from the US Department of Labor and the the US Federal Reserve and in both cases are available monthly. Time series plots for the levels of these series appear in Figure 3.

It seemed more reasonable to expect that default rates would be affected by changes in unemployment and growth in production so we expressed the two series as percentage changes over the preceding twelve months. We normalised the series by

[2]Das, Duffie, Kapadia, and Saita (2007) find some evidence that growth in US Industrial Production explains covariation in individual borrower default rates.

Figure 4: **Normalised Stress Series**

Stress series used as observable conditioning variables in the model. The series are both over the same period as the loss rate data, 1985 to 2007, and both series have been transformed to standard normal. The top panel is the annual percentage change in US Employment Rate and the lower panel is the annual percentage change in US Industrial Production.

dividing by their sample standard deviations. Figure 4 shows the normalised series employed in the stress estimation.

To estimate the parameters of the conditional default distribution, we again employed Maximum Likelihood. In a multivariate setting the weights on the observed stress factors must be normalised to ensure the $Z_{i,t}s$ have unit variance.[3]

5.3 Stress Estimates

Here, we report two univariate stresses: (i) the Credit Card loss rate conditional on the annual percentage change in the US Unemployment Rate, (ii) the C&I loss rate conditional on the same macroeconomic variable. Table 6 presents the parameter estimates for the model in the two cases.

For the Credit Card losses, the factor correlation, ρ, is 0.60%. The unconditional default probability, q, is fairly high at 1.67% and the unobservable factor reversion

[3]We performed Dicky-Fuller tests on the stress series, not reported here, that confirmed the series are stationary.

Table 6: **Univariate Stress Parameterisation**
Parameter estimates using various aggregated quarterly US bank loan loss series and stressing on the US Unemployment Rate over the period 1985 to 2007. The values in parenthesis are the t-statistics of the parameters derived from the Maximum Likelihood estimations.

	CC		CI	
Factor correlation ρ (%)	0.60	(3.83)	4.24	(1.87)
Unconditional default probability q (%)	1.67	(16.15)	0.38	(2.75)
AR(1) parameter β (%)	68.14	(8.32)	89.58	(14.57)
Stress coefficient λ (%)	-37.56	(-2.88)	-26.22	(-2.34)

parameter, β, is 68.14%. The stress coefficient, λ, is -37.56%. The sign of the stress coefficient is correct. The stress coefficient is negative. This means that as the Unemployment Rate increases, the $Z_{it}s$ decrease and so the number of defaults increases, which seems intuitive.

The C&I loss parameters are slightly different. The factor correlation, ρ, is much larger than for credit cards at 4.24%. The unconditional default probability, q, is much lower at 0.38% while the factor reversion is higher at 89.58%. Again, the stress coefficient has the correct, negative sign so the impact of conditioning on the Unemployment Rate has the expected effect.

Looking at Table 1 we can compare these results to the original estimation for the dynamic model. The parameters estimates are all slightly smaller. The correlation and default probabilities are both lower. Thus, by performing the conditioning upon an observed factor, the randomness or risk in the model is reduced.

As an example of a multivariate stress, we considered the C&I loss rate conditional on (a) the annual change in the US Unemployment Rate and (b) the annual percentage change in US Industrial Production. Table 7 shows the estimates of the model parameters for this stress case.

Comparing these results to the original estimation and the univariate stress, again the parameters are either reduced or are similar. The correlation has fallen from 4.24% to 4.18% and the unconditional default probability is the same. Hence, again by stressing on more observable factors the risk in the system has been lowered.

Thus, by sensible selection of macro-economic variables that describe an economic recession or similar cyclical event, one can condition upon these variables and assess

Table 7: **Multivariate Stress Parameterisation**

Parameter estimates using C&I quarterly US bank loan loss series and stressing on the US Unemployment Rate and US Industrial Production over the period 1985 to 2007. The stress coefficients $a_u\lambda$ and $a_{ip}\lambda$ have been normalised to ensure unit variance. The values in parenthesis are the t-statistics of the parameters derived from the Maximum Likelihood estimations.

	CI	
Factor correlation ρ (%)	4.18	(1.94)
Unconditional default probability q (%)	0.38	(3.01)
AR(1) parameter β (%)	89.38	(14.99)
Unemployment coefficient $a_u\lambda$ (%)	-23.18	(-1.92)
Industrial production coefficient $a_{ip}\lambda$ (%)	5.37	(0.56)

the risk impact. The following section discusses these ideas further.

5.4 Stressed Default Rate Distributions

To explore how the distribution of transformed loss rates is affected by conditioning upon a specific macroeconomic scenario, we performed a Monte Carlo simulation of the loss rate over a four year period. We chose an extreme period in that we conditioned on the maximum positive change in the annual percentage difference of the Unemployment Rate over a four year period observed in the sample period for which US unemployment data is available. This was the period 1950-1954 and Figure 5 shows a plot of unemployment in these years.

Figure 6 shows Credit Card loss rates simulated over a four year period for conditional and unconditional cases. Comparing the shape of these loss paths to the Unemployment Rate stress, Figure 5, one can see how the loss rates are driven by the observable factor. The loss in period four has increased dramatically on average compared to the unconditional case.

Figure 7 shows the simulated loss distributions at one- and four-year horizons. Table 8 shows statistics for these distributions. Looking first at the four year horizon, one can see how the unemployment stress has driven the Credit Card loss rate upwards. The mean loss is now 3.19% whereas in the unconditional case it is 1.69%.

The one-year horizon loss rates present a different picture. Note that in Figure 5, the Unemployment Rate actually drops in the middle of the first year before rising

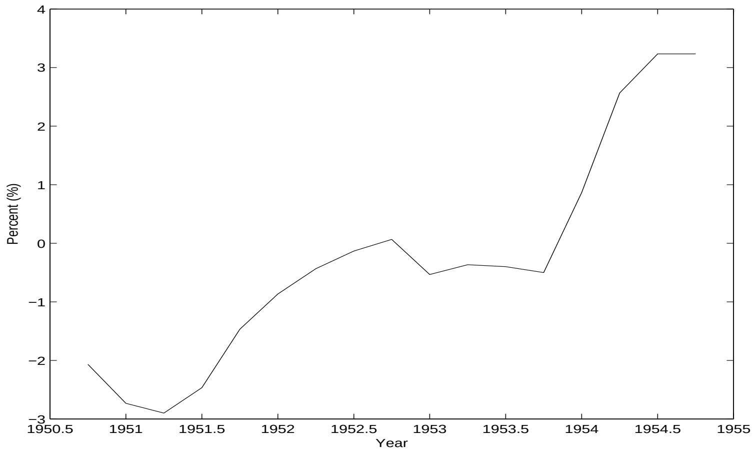

Figure 5: **Unemployment Recession Stress**
The four year period of maximum positive change in the annual percentage differences in US Unemployment Rate. Stress period found to be 1950 to 1954. The series has been transformed to standard normal using the same normalisation parameters as those in the estimation exercise.

again to be slightly greater than its value at the start period. As the Unemployment Rate has increased in value over the one-year period, the mean loss in the conditional case is higher, at 1.81%, than the unconditional case, at 1.69%. But, due to the drop in the Unemployment Rate over the first six months, this then reduces the volatility of the loss distribution.

6 Capital Implications

6.1 Implied Capital

In this section, we consider the implications of our dynamic default rates for capital modelling. Suppose that losses on a bank's portfolio in period t are a function only of the single risk factor described above, namely X_t. As shown by Gordy (2003), the Marginal Value at Risk (MVaR_α) for a single loan may be calculated as the expected loss on the exposure conditional on X_t being at its α-quantile. But the expected loss is just equal to the probability of default, $\Phi(\tilde{\theta})$, multiplied by the loss given default (LGD), i.e.,

$$\text{MVaR}_\alpha \quad = \quad \text{LGD} \times \Phi\left(\frac{\Phi^{-1}(q) - \sqrt{\rho}\sqrt{\beta}X_{t-1} - \sqrt{\rho}\sqrt{1-\beta}\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right) , \qquad (41)$$

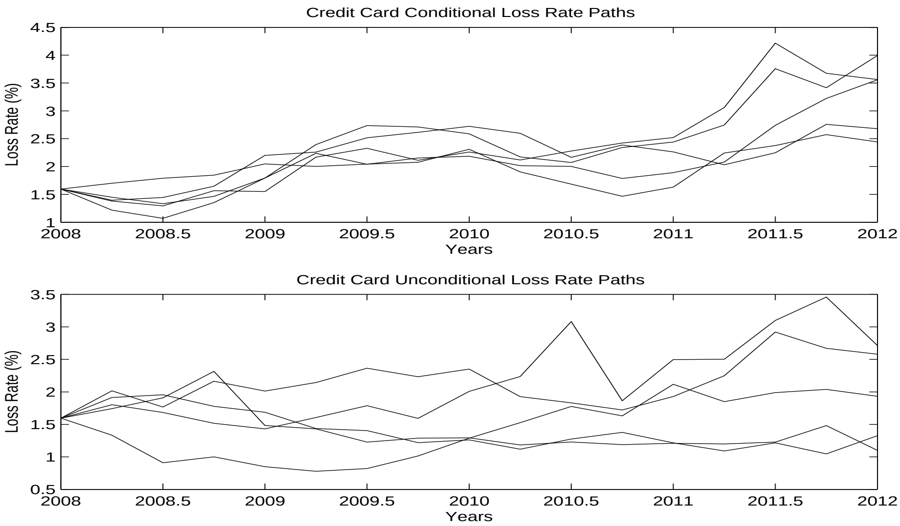

Figure 6: **Stress Monte Carlo Paths**

Example loss rate paths over a four year period for the conditional and unconditional cases of US Credit Card loss rates stressing on the annual percentage change in US Unemployment Rate between 1950-1954.

where the factor shock, η_t is taken at its α-quantile.

The probability of default conditional on information at $t-1$ is given by equation (9). Substituting yields the proposition:

Proposition 4 *Under the above assumptions, the Marginal Value at Risk for a single loan, denoted* MVaR_α*, is:*

$$\text{MVaR}_\alpha \quad = \quad \text{LGD} \times \Phi\left(\sqrt{1-\rho\beta}\frac{\Phi^{-1}(q_t) - \sqrt{\rho}\sqrt{1-\beta}\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right). \tag{42}$$

The Basel II capital formulae are based on Unexpected Loss i.e., the MVaR_α minus the expected loss:

$$\text{Basel Capital Formula} \quad = \quad \text{LGD} \times \Phi\left(\frac{\Phi^{-1}(q) + \sqrt{\rho}\sqrt{1-\beta}\Phi^{-1}(\alpha^*)}{\sqrt{1-\rho}}\right) - \text{LGD} \times q. \tag{43}$$

where α^* equals $1-\alpha$ in our notation. Note here that the Basel formula is simplified by the absence of the expression $\sqrt{1-\rho\beta}$ and depends on q rather than the conditional default probability q_t. When $\beta = 0$ so the factor is no longer autocorrelated, the latter term simplifies to unity and $q_t = q$ for all t. Since $\phi^{-1}(\alpha^*) = -\phi^{-1}(\alpha)$, in this case, the Basel capital formula equals the MVaR_α expression in equation (42).

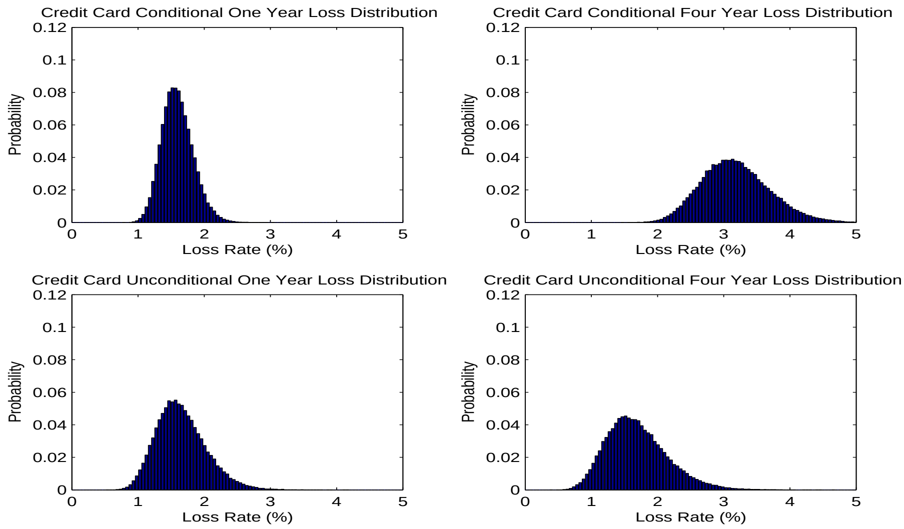

Figure 7: **Stressed Loss Distributions**
One and four year conditional and unconditional simulated loss distributions of US Credit Card loss rates stressing on the annual percentage change in US Unemployment Rate between 1950-1954.

6.2 Regulatory Implications

It is a straightforward exercise to relate capital charges as used by regulators and banks if the data period is one year. We therefore convert the data from quarterly series to their annual representations for the following capital calculations.

We calculate capital under the Basel II rules and based on MVaRs implied by the estimated processes with and without autocorrelation. The Basel II calculation assumes a point-in-time default probability as a parameter for their formulae. We take the loss rate at the end of the sample period for this parameter as this can be viewed as a proxy for the conditional default probability.

Table 9 reports the capital requirements calculated by all models. The top half of the table displays the parameters assumed for the Basel II calculation. The lower half of the table reports the capital for the three different cases. It is immediately apparent that the capital implied by the autocorrelation case is less than the static case for all series. For the Real Estate category, the implied capital for the autocorrelation case is 0.61%, this is approximately 5 times less than the static case at 2.35%. Other series are, in relative terms, closer however. The credit card calculation implies 3.87% in the static case and 2.16% in the autocorrelation case.

Table 8: **Stressed Loss Distribution Statistics**

Simulated US Credit Card loss distributions over quarterly periods to 4 years, stressed and unstressed. In the conditional case the annual percentage differences in US Unemployment Rate over the period 1950 to 1954 were used. An initial loss rate of 1.60% was assumed.

	Conditional		Unconditional	
	Year 1	Year 4	Year 1	Year 4
Mean	1.81	3.19	1.66	1.69
Median	1.79	3.15	1.62	1.64
90% quantile	2.19	3.86	2.20	2.30
95% quantile	2.31	4.09	2.39	2.53

The Basel II capital comparisons are even more striking. In all cases with or without autocorrelation the implied capital is substantially less than what Basel suggests. As an example, the other consumer category implies 1.11% using the autocorrelation model, 2.09% with the static model and 8.26% under the Basel II rules. In all cases the order of magnitude is at least four times greater for the Basel II calculation than the autocorrelation model and in some cases the difference is even greater.

This is not surprising in the case of long-lived assets like real estate, leases and C&I as the Basel parameterization in these cases is based on economic-loss-mode MVaRs from ratings-based models like Creditmetrics. (Default mode capital curve formulae are just convenient functions that happens to have the right shape.) But for short term assets including retail (especially credit cards), this difference in the implied capital can be viewed as very important for banks and financial institutions that hold such books.

7 Conclusion

This paper presents a generalization of the widely used Vasicek model of loan losses. Our generalization allows for the dynamic evolution of loan loss distributions. Introducing dynamics is essential if such models are to be employed to analyze real life data since loan losses exhibit clear signs of autocorrelation and trends.

The analysis provides a framework and a set of tools for examining portfolio credit risk at an aggregate level. The results show that the risk characteristics and general behaviour of losses in a conditional and unconditional world are very different.

Table 9: **Capital Calculations**

Capital calculations at the 99.9% loss quantile comparing Basel II implied capital against a static model and a model with factor autocorrelation for different US bank loss rates over a one year period. The capital is calculated using equation (42). The common parameterisation is shown in the top panel of the table. The LGDs are taken from Basel Committee on Banking Supervision (1999). The factor correlations used are either the Basel II implied correlations or estimated using an annual model. In the model with autocorrelation, the autocorrelation coefficient is also shown.

	RE	CC	OC	L	CI	A
Common Parameters						
Default probability (%)	0.63	5.95	2.37	0.53	1.08	0.21
Loss given default LGD	0.35	0.65	0.65	0.45	0.45	0.45
Factor Correlation Estimates ρ (%)						
Basel II	15.00	4.00	8.66	21.22	19.00	22.83
With no autocorrelation	10.53	1.31	1.33	5.16	7.25	13.98
With autocorrelation	10.91	0.88	1.43	5.22	7.17	6.84
AR(1) parameter β (%)	80.07	44.82	66.68	41.33	65.03	59.15
Capital Calculations (%)						
Basel II	3.37	7.97	8.26	5.70	7.59	3.56
With no autocorrelation	2.35	3.87	2.09	1.38	3.00	1.98
With autocorrelation	0.61	2.16	1.11	0.89	1.28	0.42

Linking models to macro-economic variables or indicators is a simple task using this model. This can help banks or other financial institutions analyse their risks in a dynamic sense and allow stress testing of business cycle effects.

Appendix

A Eigenvalue Tests for a Covariance Matrix

This appendix discusses how to test for the importance of eigenvalues in the decomposition of a covariance matrix. The testing hypothesis is taken from Anderson (2003).

Suppose that the sum of the last $p - m$ eigenvalues of a decomposed covariance matrix are small compared to the sum of all the roots of the covariance matrix. A null hypothesis of the form:

$$H : f(\lambda) = \frac{\lambda_{m+1} + \cdot + \lambda_p}{(\lambda_1 + \cdots + \lambda_p)^2} \geq \delta, \tag{A1}$$

where δ is a specified confidence level, could then be defined to reject the hypothesis that these roots are negligible.

The null is then rejected if:

$$\sqrt{n}(f(\lambda) - \delta) \leq \Phi^{-1}(1 - \delta)\sigma_\lambda, \tag{A2}$$

where Φ is the standard normal cdf and the variance of $f(\lambda)$ is given by:

$$\sigma_\lambda^2 = 2\left(\frac{\delta}{\sum_{i=1}^{p} \lambda_i}\right)^2 \sum_{i=1}^{m} \lambda_i^2 + 2\left(1 - \frac{\delta}{\sum_{i=1}^{p} \lambda_i}\right)^2 \sum_{i=m+1}^{p} \lambda_i^2. \tag{A3}$$

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