The Simplified Arbitrage-Free Approach:
Calculating securitisation capital based on Risk Weights alone

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Abstract

This paper proposes a variant of the Arbitrage Free Approach (AFA) (developed by Duponcheele et al. (2013)) applicable when the available inputs are risk weights alone. This Simplified AFA, together with the AFA itself, offer a consistent set of approaches for regulatory capital calculations that can be used by investor or issuer banks without relying on agency ratings. The consistency of the AFA and SAFA has the significant advantage of eliminating the cliff effects that occur with the Modified Supervisory Formula Approach (MSFA) and the Simplified Supervisory Formula Approach (SSFA) recently suggested by the Basel Committee. The latter are based on different assumptions and so may imply quite different capital for a given securitisation tranche.

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SECTION 1 - INTRODUCTION

The Basel Committee's recently published proposals for securitisation regulatory capital include several approaches that place different informational demands on users. The most elaborate approach, the Modified Supervisory Formula Approach (MSFA) (set out in BCBS (2012) and detailed in BCBS (2013a)), requires, as inputs, detailed exposure level information on the securitisation pool. The information required includes the default probability (PD_i), mean loss given default (LGD_i), exposure at default and the systemic sector correlation (ρ_i) (dependent on its regulatory asset class) for each individual asset.

In practice, it is unrealistic to expect all banks calculating securitisations capital to have access to certified estimates of the Basel parameters (i.e PD_i, LGD_i and ρ_i) for each individual asset. It is even more unrealistic to expect that they will all be able to estimate these quantities, especially when they are not originators, while satisfying the rigorous data standards of the Internal Ratings Based Approach (IRBA).

In particular, investor banks (whether IRBA or Standardised Approach (SA) banks) typically have relatively little detailed information of the type required by regulators on underlying pool assets. Even when they have data on the majority of a pool's exposures, having less complete information on a minority of assets would be sufficient to preclude use of the MSFA.

Furthermore, even for originating banks under the IRB approach, for certain regulatory asset classes, capital is specified directly in the form of a risk weight, RW_i, and in such cases there is no possibility of accessing PD_i or LGD_i or ρ_i separately. For example, under the Basel 2 supervisory slotting criteria for the Project Finance asset class (which includes infrastructure financing, key to the real economy), or for other Specialised Lending assets class, the banks apply directly a risk weight, RW_i.[3]

For these reasons, in their proposals, the Basel authorities have supplemented the MSFA with the Simplified Supervisory Formula Approach (SSFA). Use of the SSFA requires that users know the total regulatory capital under the so-called Standardised Approach that pool exposures would attract if they were held on balance sheet (i.e., the so-called K_{SA}[4]).

Finding its roots in BCBS (2001), the SSFA was later developed and applied in the United States for trading book securitisations. The SSFA allocates capital to different tranches of a securitisation using a simple exponential smoothing. Tranches are allocated capital equal to their par value if their detachment point is less than K_{SA}. If the attachment point is greater than K_{SA}, they are assigned a fraction of K_{SA} determined by an exponential smoothing function that allocate more to mezzanine than to senior tranches.[5] The exponential weighting function employed, though reasonable, is ad hoc and has no theoretical justification.

[3] In this case, even if IRB banks were to have access to default probabilities and loss given default means, no supervisory systemic asset value correlation ρ_i is available. An infrastructure asset in a securitisation pool precludes use of a formula that requires the 3 Basel parameters (PD_i,LGD_i,ρ_i) for each underlying pool asset.
[4] An adjustment is made for delinquent assets.
[5] If K_{SA} lies between the attachment and detachment points, a weighted average of these two approaches is employed.

As already mentioned, the SSFA has been applied in the United States for calculating trading book capital for securitisations. In that application, the calibration employed implies that total capital for all tranches in a securitisation equals 1.5 times the level of capital the pool assets would attract if held by the bank on balance sheet under the Standardised Approach. In the new use now envisaged by the Basel Committee (BCBS (2012)), namely for banking book securitisation capital, the ratio has been boosted to 2.5 times. (Note that for an IRB bank the ratio of pre- to post securitisation capital would be still greater because the Standardised Approach calculation for on-balance-sheet assets is, in many cases, more conservative than the corresponding IRB calculation.) The main motivation for calibrating the SSFA to imply a multiple of 2.5 times is apparently that it is then in most cases more conservative than the MSFA.

There is a widespread industry perception that the capital levels implied by the MSFA are higher than is economically justified, especially for long maturity securitisations. To arrive at its very conservative capital charges, the MSFA includes expected losses as well as unexpected losses in its definition of capital. The expected losses are not the regulatory definition of expected losses used in IRB, but instead are calculated inclusive of a conservative risk premium and cover a period up to the minimum of (i) five years or (ii) the contractual maturity of the deal. While the SSFA does not include an explicit maturity adjustment, the degree of conservatism it embodies is also extremely high.

The fact that the MSFA and the SSFA are not derived from a coherent, unified underlying theory implies that the levels of capital they imply are only roughly comparable. Small changes in the availability of data on the underlying pool that oblige a bank to use the SSFA on the entire pool instead of the MSFA may, therefore, imply a sizeable jump in capital levels, i.e., a "cliff effect"[6].

As an alternative to the MSFA, Duponcheele, Perraudin and Totouom-Tangho (2013) propose a closed form solution for calculating the capital of securitisation tranches, known as the 'Arbitrage-Free Approach' (AFA). In order to avoid capital arbitrage when portfolios exhibit heterogeneity and/or low granularity, Duponcheele et al. (2013) set out an approach (termed 'Option 2' in the paper). To implement the AFA, a bank must know the 3 Basel parameters (PD_i, LGD_i, ρ_i) associated with each underlying asset i in the pool.[7]

'Option 2' was developed in recognition of the fact that allowing banks to base capital on a single number, RW_{Pool}, may permit them to engage in capital arbitrage, in particular by securitising extreme "bar-bell" type pools[8]. While it is important to reduce scope for this type

[6] The term 'cliff-effect' is used in the context of jumps in capital requirements. Within securitisation, there are 3 broad categories of jumps. Firstly, in the AFA paper, we refer to the term 'cliff-effect' in the context of a mathematical discontinuity caused by the use the ASRF model, designed for diversified banks, to determine the capital requirement of concentrated securitised portfolios. Secondly, in this paper, we use the term 'cliff-effect' to indicate that a change in the applicable formula due to a small change in available regulatory information, for the same underlying capital, creates large jumps in the capital requirement of the entire securitised portfolios. Thirdly, in the context of the RBA, the term 'cliff-effect' refers to the fact that senior tranches are capital deducted when their rating drops to below BB-, requiring in the process several multiples of the capital requirement of the underlying portfolios.

[7] A step-by-step implementation of 'Option 2' is provided in Appendix 6 of Duponcheele et al (2013).

[8] By assuming random attachment points, the SFA formula attempts to rectify the cliff effect in capital that follows from the assumption of a single asymptotic risk factor driving both bank portfolio and securitization pool. The random attachment point device reduces but does not eliminate the cliff effect, however, and, as is

of capital arbitrage, it is worth noting that such aggressive structures represent a minuscule part of the securitisation market and are tackled anyway by regulators exercising their judgement when faced with such transactions[9].

Just as the MSFA is accompanied in the Basel proposals with a less informationally demanding alternative, namely the SSFA, one may ask how one might simplify the AFA to permit banks to calculate capital without full information on the regulatory PD_i, LGD_i and ρ_i of each individual asset[10]. Answering that question is the topic of this paper.

Building on the simple closed form 'Option 1' described in Duponcheele et al (2013), in the sections below, we present a capital formula designed for handling securitisation tranches when the level of information on pool assets consists of Standardised Approach risk weights. The resulting approach is termed the 'Simplified Arbitrage-Free Approach' or SAFA.

As mentioned above, a simplified approach is particularly important for investor/sponsor/dealer banks. Taking as input a single number[11] that summarises underlying pool capital, a simplified approach would enable banks to calculate the capital requirements of tranches without having to rely on rating agencies. This is an obligation under Dodd-Frank and is in line with a stated objective of BCBS (2012): to reduce reliance on rating agency credit assessments.

If the AFA framework and the SAFA framework were both made available to banks, the problems around so-called mixed pools (in which IRB parameters are available for some exposures while, for others, only Standardised Approach information is available) could be addressed. In this case, using the 'Option 2' of Duponcheele et al (2013), one could calculate capital for the two types of exposure by working out their respective capital contributions, either with the AFA or with the SAFA, to notional structures with theoretical homogeneous sub-pools. In this way, both heterogeneity and mixed pool issues can be managed at the same time.

The rest of the paper is organised as follows. Section 2 analyses the SSFA and shows in what respects it deviates from the four principles for devising securitisation capital approaches discussed in Duponcheele et al (2013). Section 3 derives a formula for securitisation capital under our proposed SAFA and discusses a further simplification of the SAFA undertaken as to make it monotonic[12], the Monotone SAFA (MSAFA). Section 4 presents illustrative calculations using different capital formulae. Section 5 analyses the link between the p

well known, capital relief arbitrage may be obtained by securitising bar-bell portfolios. In this case, the 'good' granular credits generate the granularity necessary to flatten the SFA distribution, ensuring that the 'bad' credits receive relatively favourable treatment.

[9] Some regulators have actually disallowed the use of the SFA on specific transactions and have required banks to use the RBA instead.

[10] Note that the regulatory requirements on PDs and LGDs under the IRBA framework are much more demanding than simply estimating a PD and LGD for each position.

[11] The current SFA formula takes as input the aggregate pool capital (K_{IRB} including expected loss). The minimum IRB requirement standards require that, to calculate $K_{IRB,}$ banks have access to data standards enabling them to compute regulatory PD_i and LGD_i for each underlying asset. If this data were to be made available to investors, the SFA would be quite akin to using a single risk weight number for the allocation of capital to the tranches.

[12] By 'monotonic', we mean that the risk weight of a more junior tranche is always greater or equal to the risk weight of a more senior tranche, regardless of its thickness.

parameter of the SSFA and the ρ^* parameter of the SAFA. Section 6 concludes. The Appendix provides details of how to implement the SAFA and MSAFA and additional technical material.

SECTION 2 – IS THE SSFA A SIMPLIFICATION OF THE SFA?

Contrary to a common view, the Simplified Supervisory Formula Approach (SSFA) is not a simplified version of the existing SFA. Instead, it is a direct descendant of the original Supervisory Formula (SF), presented in the Basel Committee's October 2001 working paper on securitisation capital (see BCBS (2001)).

In that document, the following Supervisory Formula was proposed. Let T_1 and T_2 denote the attachment and detachment points of a securitisation. Let K_{IRB} denote the reference capital level (including expected loss) on the underlying pool. For a premium factor, β, and a parameter δ satisfying the equation $1/\delta\left[1 - e^{(K_{IRB}-1)\delta}\right] = \beta K_{IRB}$, the capital requirement denoted CR_T of a tranche attaching above K_{IRB} is given by:

$$CR_T = 1/\delta \cdot \left[e^{(K_{IRB}-T_1)\delta} - e^{(K_{IRB}-T_2)\delta}\right] \tag{1}$$

The parameter β is here called the 'premium' factor while the value of δ determines the distribution of total capital (equal to $(1+\beta)\ K_{IRB}$) across tranches.

An important question when the SF was first proposed concerned the appropriate size of the premium β and the Basel Committee proposed a calibration of 20%. This ensured that the total capital of all the tranches equalled 20% more than the capital the assets would attract prior to securitisation. Pykhtin and Mingo (2002) analysed the Basel 2001 proposal by analysing the correlation between the bank's portfolio and the securitised assets. They argued that an appropriate value of β lies in the 3%-11% range[13].

The exponential smoothing of the additional capital βK_{IRB} (over and above K_{IRB}) in the SF was not based on a theoretical model but reflected the belief that capital should be concentrated on the mezzanine tranches rather than the senior tranches. The exponential function was a simple function that accomplished this. The fact that the exponential function converges rapidly to zero, however, implied extremely low capital for the more senior tranches, and, in a subsequent paper (see BCBS (2002)), the Basel Committee revised their earlier SF proposal by including a floor capital level.

Furthermore, the original SF was superseded in subsequent Basel proposals by the Supervisory Formula Approach. This was derived from an explicit theoretical model in which the credit quality of exposures in both the bank portfolio and the securitisation pool are assumed to be driven by the same Single Asymptotic Risk Factor. The extreme cliff effect that this assumption implies for mezzanine tranches was mitigated by the somewhat questionable device of assuming that attachment points of tranches are random.

[13] Their conclusion is consistent with analysis of the link between the p parameter of the SSFA and the ρ* of the AFA in Section 6 below.

In the latest incarnation of the SF, i.e., in the SSFA, the capital that is distributed by the formula is the Standardised Approach capital aggregate, K_{SA} (or, when there are delinquent assets in the pool, an adjusted version of K_{SA} known as K_A) rather than the K_{IRB} which was distributed by the original SF.[14] The parameter δ has been simplified in that there is a new parameter a satisfying $a = \frac{-1}{p\,K_A}$. Here, p plays a role analogous to β in that it determines the addition to total capital when one goes from pre- to post-securitisation.

Under the SSFA, for a given tranche (with an attachment and detachment points, A and D, that exceed K_A), capital for a given tranche equals:

$$K_{SSFA}(L,U) = \frac{(e^{aU} - e^{aL})}{a(U-L)} \tag{2}$$

Here, $U = D - K_A$ and $L = \max(\mathrm{A} - K_A, 0)$.[15]

It is interesting to evaluate the SSFA against the principles that should guide the development of securitisation capital formulae according to Duponcheele et al (2013). These principles are:

1. Objective statistical basis: Capital for securitisation exposures should be based on their marginal contribution to a single, widely accepted statistical measure of the bank's total portfolio risk.
2. Neutrality: Apart from model risk or agency effect add-ons, the capital a bank must hold against a set of assets should be unaffected by repackaging these assets into other securities as is achieved through securitisation.
3. Regulatory control: Control parameters should be available that permit regulators and supervisors to achieve their objectives and exercise judgments in the allocation of capital across different types of exposure. Such parameters should reflect the economic reality of transactions so that they could in principle be calibrated from empirical data.
4. Transparency: Capital formulae should reflect in a simple way the nature of risk and be consistent with other regulatory capital approaches to facilitate comparisons and to promote transparency.

Table 1 presents an evaluation of how the SSFA measures up to these principles.

Table 1: Evaluation of the SSFA

Principle	How the Simplified Supervisory Formula Approach Performs
1. Objective statistical basis	The SSFA does not change the statistical criterion for risk for the underlying assets since it employs the underlying capital as an input. It is risk sensitive (if we leave aside the deduction below K_A and the floor). The SSFA may be regarded as a distributive function.

[14] Specifically, if a fraction of the pool, W is delinquent, then, $K_A = (1-W)K_{SA} + 0.5\,W$. A pool asset is defined to be delinquent if it is "90 days or more past due, subject to a bankruptcy or insolvency proceeding, in the process of foreclosure, held as real estate owned, had contractually deferred interest payments for 90 days or more, or were in default." (See BCBS (2012), page 22.)

[15] When, $< K_A$, the tranche Risk Weight is 1250%. When $A > K_A$, the Risk Weight is 12.5 times K_{SSFA}. When $D < K_A < A$, the Risk Weight is set equal to: $12.5\frac{K_A - A}{D-A} + 12.5K_{SSFA}\frac{(D-K_A)}{(D-A)}$.

2. Neutrality	The SSFA could easily be modified to enforce neutrality by requiring deduction up to $K_A(1-p)$ and exponential interpolation above this value of the additional capital, pK_A. In this case, a value of $p = 15\%$ would, overall, yield results similar to the AFA[16] with conservative levels of concentration correlation ρ^*. Instead, a large boost in capital is proposed. A value of $p = 50\%$ implies that the total capital of a securitisation is 50% greater than the capital the assets would attract if held on balance sheet by a Standardised Approach bank. A value of $p = 50\%$ also implies a high dispersion of capital across senior tranches. Such an approach might be justified to allow for including market and agency risks, say in the context of US Subprime RMBS held in the trading book. But, it appears too conservative when applied to European banking book securitisation exposures for which agency risk is likely to be much less. A value of $p = 150\%$ implies that capital after securitisation is 2.5 higher than before. When combined with the exponential smoothing, viewed from the perspective of the AFA, this would imply an unreasonable degree of correlation across pool exposures. An '*overall-cap*' (even for the portion above K_A) is required to avoid the capital levels becoming absurd. With the calibration currently proposed, the SSFA turns fundamentally away from the notion of neutrality in its construction, even though neutrality could easily be achieved.
3. Control parameters	The SSFA has a parameter p which gives the percentage of additional capital that is needed above an attachment point equal to K_A. As far as we are aware, no empirical evidence has been given by the Basel Committee to justify a level of $p = 50\%$ or $p = 150\%$. The work done by Pykhtin and Mingo (2002) indicates a level below 15%.
4. Transparency	The SSFA is transparent. The parameter p can be easily interpreted as a 'premium', but it is disconnected from an economic description of the risk of the underlying assets.

To conclude, the capital charge formulae presented in BCBS (2012) for the SSFA is derived from the original Supervisory Formula (SF) of the initial Basel proposal for securitisation (BSBS (2001)). The SSFA adopts a level and distribution of capital above K_A which is hard to justify empirically. When $p = 150\%$, caps are required to mitigate the unreasonable outcomes that arise. Another approach, as simple and transparent as the SSFA, that respects the principles listed above would be preferable.

SECTION 3 – THE SIMPLIFIED AFA (SAFA)

In this section, we set out a simplified version of the AFA developed by Duponcheele et al. (2013). The simplifications take the form of permitting inputs that demand less regulatory information about the underlying assets. The approach is sufficiently simple that it could serve as the basis of capital calculations for banks that typically have little data on pools and where such data is available, it is often not of the quality required in the IRBA rules for on-balance-sheet capital calculations. The Simplified AFA or SAFA we describe could serve as a substitute for the use of external agency ratings.

[16] Although results of such SSFA with p=0.15 would probably be correct for the most senior tranches and most junior tranches, they would remain inadequate for mezzanine tranches.

The AFA is stated in Duponcheele et al (2013) as follows, with A being the attachment point and D the detachment point of the tranche:

$$Capital = EL_{Thick}(A, D, PD'_{\alpha}, \rho^*, LGD) - EL_{Thick}(A, D, PD', \rho_{\text{Pool}}, LGD) + MRC \qquad (3)$$

Where, depending on the IRBA asset class:

$$\rho_{\text{Pool}} = \rho_{Sector} + (1 - \rho_{Sector})\rho^*$$

$$N^{-1}(\alpha) = -N^{-1}(99.9\%)$$

$$PD_{\alpha}(PD) = N\left(\frac{N^{-1}(PD) - \sqrt{\rho_{Sector}}\, N^{-1}(\alpha)}{\sqrt{1-\rho_{Sector}}}\right)$$

$$PD' = PD \times MatAdj(M, PD)$$

$$PD'_{\alpha} = PD_{\alpha} \times MatAdj(M, PD)$$

$$MRC = 6\% \times \left(PD'_{\alpha} \cdot LGD - PD' \cdot LGD\right)$$

$$EL_{Thick}(A, D, P, \rho, LGD) = \frac{(1-A)}{D-A} EL_{SeniorTranche}(A, P, \rho, LGD) - \frac{(1-D)}{D-A} EL_{SeniorTranche}(D, P, \rho, LGD)$$

$$EL_{SeniorTranche}(X, P, \rho, LGD) = \frac{LGD \times \overline{N}_2(X, P, \rho, LGD) - X \times PD_{Tranche}(X, P, \rho, LGD)}{1-X}$$

$$\overline{N}_2(X, P, \rho, LGD) \equiv N_2\left(N^{-1}(P), N^{-1}\left(PD_{Tranche}(X, P, \rho, LGD)\right), \sqrt{\rho}\right)$$

$$PD_{Tranche}(X, P, \rho, LGD) = N\left(\frac{N^{-1}(P) - \sqrt{1-\rho} \cdot N^{-1}\left(\frac{X}{LGD}\right)}{\sqrt{\rho}}\right)$$

$$MatAdj(M, P) = \left(\frac{1 + (M - 2.5)(0.11852 - 0.05478\ln(P))^2}{1 - 1.5(0.11852 - 0.05478\ln(P))^2}\right) \quad or\ 1\ depending\ on\ the\ IRBA\ asset\ class$$

(4)

The maturity adjustment $MatAdj$ is equal to that currently used in the IRBA on-balance-sheet loan rules.

The above formulae are derived for a securitisation with a homogeneous pool. Using pool-level information, Duponcheele et al (2013) set out how to apply the approach to cases with heterogeneous pools (in their 'Option 1') by replacing, in the above formulae, PD'_{α} with $(K_{IRB} + EL')/LGD$ where K_{IRB} is the Basel II pool asset capital inclusive of maturity adjustment, EL' is the total pool expected loss inclusive of maturity adjustment and LGD is the total estimated mean loss for the pool assets. Similarly, PD' is replaced with EL'/LGD.

Using individual loan-level information, for heterogeneous pools, Duponcheele et al (2013) suggest (see their 'Option 2') calculating capital contributions for individual pool exposures based on the capital that would be implied by a notional homogeneous pool securitisation tranche with ρ^* and $\rho_{i,Pool}$ replaced with granularity-adjusted versions[17].

Since calculating the inputs for both Options 1 and 2 of Duponcheele et al (2013) requires access to detailed individual pool exposure level information, the list of inputs is:

PD_i	Asset default probability for each exposure
LGD_i	Asset loss given default for each exposure
ρ^*	Concentration correlation
$\rho_{Sector,i}$	Systemic sector correlation for each exposure
M_i	Asset maturity for each exposure
EAD_i	The exposure weight of each exposure

Suppose that, instead, either (i) the risk weights (RWs) rather than the PDs and LGDs of each asset or (ii) the average PD and LGD of the pool are available, one may 'reverse engineer' the relevant inputs. Specifically, one may proceed (as we shall describe below) by inferring the following parameters:

PD_α	Stressed pool default probability
PD	Pool default probability
LGD	Pool mean loss given default
ρ^*	Pool concentration correlation[18]
ρ_{Sector}	Pool systemic sector correlation

One may then employ them as inputs to the equation (5) with PD_α being a parameter.

$$Capital = EL_{Thick}(A, D, PD_\alpha, \rho^*, LGD) - EL_{Thick}(A, D, PD, \rho_{\text{Pool}}, LGD) + MRC \tag{5}$$

To infer the above AFA input parameters from risk weights RWs, first, note that conditional on Y_{Bank} (the systemic risk factor) equalling its 99.9% confidence level, the default probability of pool exposures is given by:

$$PD'_\alpha = \frac{UL_{Pool} + EL_{Pool}}{LGD_{Pool}} \tag{6}$$

Here, UL_{Pool} is the total unexpected loss for the pool of assets (better known as the Basel II IRB capital for the pool assets), EL_{Pool} is the total regulatory expected loss for the pool of assets (inclusive of maturity adjustment) and LGD_{Pool} is the weighted average estimated mean loss given default for the pool assets.

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[17] These are respectively: $\rho^* + \delta_i(1-\rho^*)$ and $\rho_{i,Pool} + \delta_i(1-\rho_{i,Pool})$. In the current context, i.e., in an Option 2 calculation, the δ_i should be calculated on a consolidated obligor basis as: EAD_i / EAD_{Pool}. Note that, in contrast, in an Option 1 calculation, it is appropriate to calculate the adjustment based on the approach used in BCBS (2006), paragraph 633, i.e., based on: $\sum EAD_i^2 / \left(\sum EAD_i\right)^2$.

[18] Again, granularity adjustments may be added to the correlations as described in footnote 14 and in the Appendix of Duponcheele et al (2013).

Returning to how one may proceed given risk weight RW alone, one may define:

$$UL_{RW} = \frac{RW}{12.5 \times MRSF} \tag{7}$$

We suppose here that the risk weight RW makes allowance for model risk through the inclusion of a scaling factor. In the IRB approach, the scaling factor is 1.06[19]; in this case, $MRSF$, the Model Risk Scaling Factor would also be equal to 1.06.[20]

To implement the AFA when only risk weights RW are known, one may replace the conditional probability of default employed as an input in the AFA (under Option 1) with:

$$PD'_{\alpha} = \frac{UL_{RW} + EL_{RW}}{LGD_{RW}} \tag{8}$$

If the value LGD_{RW} is not known, it may be supplied by the regulators in the form of a look-up table using broad regulatory asset categories. For example, for corporates, the current IRB-Foundation approach defines regulatory mean LGDs as shown in Table 2.

Table 2: Mean LGD Parameters to use for LGD_{RW}

Framework	Type	LGD_{RW}
Wholesale – Corporate, Sovereign, Bank	Senior Claim[21]	45%
Wholesale – Corporate, Sovereign, Bank	Junior Claim	75%
Retail –Mortgages[22]	Prime	25%
Retail –Mortgages	Non-Prime	45%
Retail – Qualifying Revolving	All	85%
Retail – Other Retail	All	85%
Others	All	45%

When the value EL_{RW} is not known, it may be set by the regulators, as an add-on to boost the distribution of capital assigned to the senior tranches. Such a function could be written as:

$$EL_{RW} = p_{RW} \times UL_{RW} \tag{9}$$

Here, pool level expected losses are set equal to a fixed proportion of the risk weight of the pool, where the weight p_{RW} is set by regulators for the broad regulatory asset categories.

BCBS (2006) gives an illustration (paragraph 377) on how to link a supervisory expected loss to a supervisory risk weight. In this case, the proportion is: $p_{RW} = 8\%$. In effect, such a value

[19] In the IRB Approach, the capital requirement is the UL, increased by 6%, via the scaling factor.

[20] In the Standardised Approach the floor for AAA securities is a 20% RW (calibrated with an underlying corporate portfolio with 100% RW); this would be equivalent to setting the Model Risk Scaling Factor equal to 1.25. This reflects the calculation 100% x (1-1.25)/1.25 = 20%. In other words, by setting an MRSF of 1.25, this could give a model risk charge (equivalent to a risk sensitive floor) of 20% RW for large corporate underlyings (100% RW), 30% RW for SME underlyings (150% RW) and 7% for residential mortgages underlyings (35% RW).

[21] As in IRB Foundation Approach (see paragraphs 287 and 288, Basel (2006)).

[22] This is based on examples provided in Basel (2006), Annex 5.

would imply that the marginal value at risk of the asset is 1.08 times the UL_{RW}. In other words, we distribute with ρ^*, 1.08 times the RW of the asset.[23]

Given UL_{RW}, LGD_{RW}, and EL_{RW} as defined by knowledge of risk weights RW and the use of a look-up table such as those in Table 2 and an approach to inferring expected losses as in equation (8), one may infer PD_α as the key input to the capital formula.

To calculate tranche-level expected losses (i.e., the second term in the bracket in equation (5)) requires that one know the pool correlation, ρ_{Pool}. As shown in equation (4), in the AFA model, the intra-pool correlation is $\rho_{Pool} = \rho_{RW} + (1 - \rho_{RW})\rho^*$, where ρ_{RW} is analogous to ρ_i, the systemic correlation in the IRB approach. To derive it, it requires identifying the systemic correlation ρ_{RW} appropriately.

For some asset classes, such as residential mortgages or qualifying retail exposures, this determination is straightforward as the correlation takes a fixed value for all individual exposures. The choice is more difficult when, under the Basel assumptions, the individual exposure correlation depends on the probability of default of the assets. This default probability is assumed to be unavailable to a bank implementing the SAFA. To be broadly compatible with BCBS (2006), we suggest that a lookup table be employed including values such as those set out in Table 3.

Table 3: Correlation Parameters

Framework	**Type**	$\boldsymbol{\rho_{RW}}$
Wholesale	Corporate, Sovereign, Bank [24]	21%
Wholesale	SME[25]	10%
Wholesale	Other[26]	12%
Retail – Mortgages[27]	Prime/Non-Prime	15%
Retail – Qualifying Revolving[28]	All	4%
Retail – Other Retail[29]	All	10%
Others	All	12%

[23] However the value of p_{RW} is sensitive to the underlying parameters. Below are examples of typical regulatory asset classes, designed to enable a comparison between the IRB and Standardised approaches:

	PD_i	LGD_{RW}	EL_i	ρ_{RW}	UL_i	$RW_i\ (IRB)$	$RW\ (SA)$	p_{RW}
Large Corporates	1.25%	45%	*0.6%*	21%	*7.9%*	*98%*	100%	*7%*
Residential Mortgages	1.00%	25%	*0.3%*	15%	*2.8%*	*33%*	35%	*9%*
SME Corporates	3.30%	75%	*2.5%*	10%	*13.6%*	*148%*	150%	*18%*

It can be seen that a value of $p_{RW} = 8\%$ is adequate for large corporates and residential mortgages but probably insufficient for SMEs, where $p_{RW} = 20\%$ is a more appropriate order of magnitude.

[24] See Basel (2006), paragraph 272. Normally, the values for individual exposures lie between 12% (for low credit quality assets) and 24% (for high credit quality assets). A broad 'BBB-' portfolio would give: $\rho_{RW} = 21\%$.
[25] See Basel (2006), paragraph 273. The values required by the Basel rules are between 0% and 4% below the corresponding corporate correlation. Hence, a reasonable value would be 2% below 12% (the value for low credit quality corporates), ie.: $\rho_{RW} = 10\%$.
[26] The category "Other" would include specialised lending for example.
[27] See Basel (2006), paragraph 328.
[28] See Basel (2006), paragraph 329.
[29] See Basel (2006), paragraph 330. Normally, the Basel rules require values between 3% (low credit quality assets) and 16% (high credit quality assets). A reasonable "middle-ground" value would be: $\rho_{RW} = 10\%$.

For exposures under the wholesale framework, the granularity adjustment would be calculated as in paragraph 633 of BCBS (2006), with N_c being the effective number of consolidated exposures. In this case:

$$\delta_{RW} = \frac{1}{N_c} \qquad \text{where} \qquad N_c = \frac{\left(\sum_{i,i\subset c} EAD_i\right)^2}{\left(\sum_{i,i\subset c} EAD_i^2\right)} \tag{10}$$

For securitisation pools comprising retail assets only, granularity is very high, and there is no need to include a granularity adjustment, therefore, $\delta_{RW} = 0\%$.

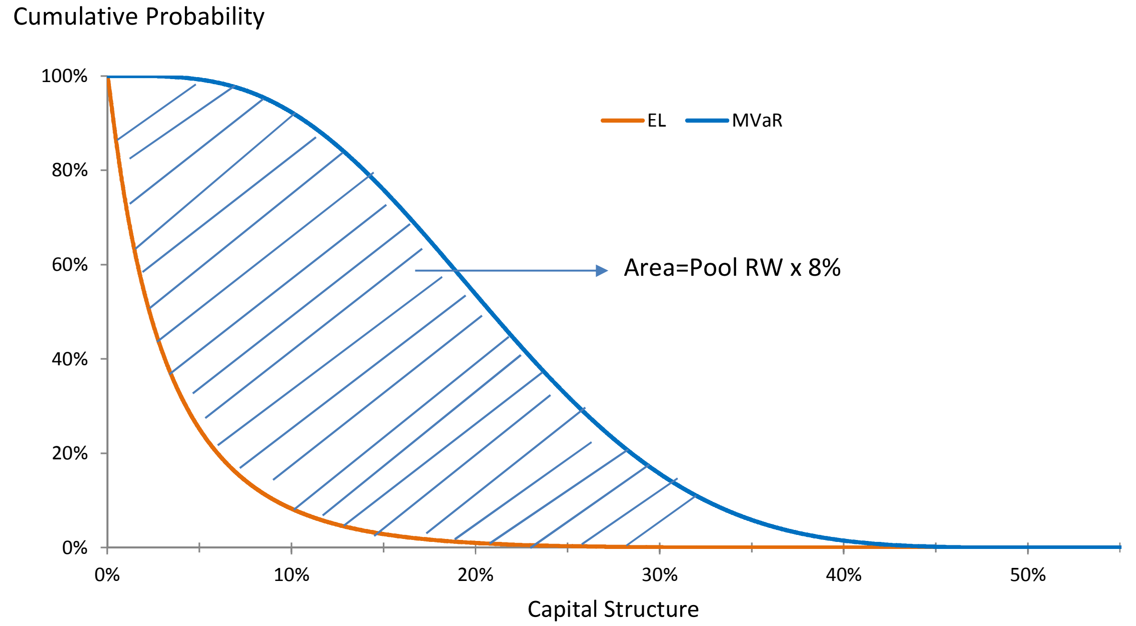
For the wholesale framework, if the effective number of consolidated exposures can be shown[30] to be greater than 100, then $\delta_{RW} = 0\%$, otherwise $\delta_{RW} = 1/N_c$.

Table 4 summarizes the granularity adjustments we propose.

Table 4: Granularity Adjustments

Framework	Type	δ_{RW}
Wholesale	Corporate, Sovereign, Bank[31]	0% if $N_c > 100$. Otherwise $\frac{1}{N_c}$
Retail	All	0%

Figure 1: Pool Risk Weights[32]

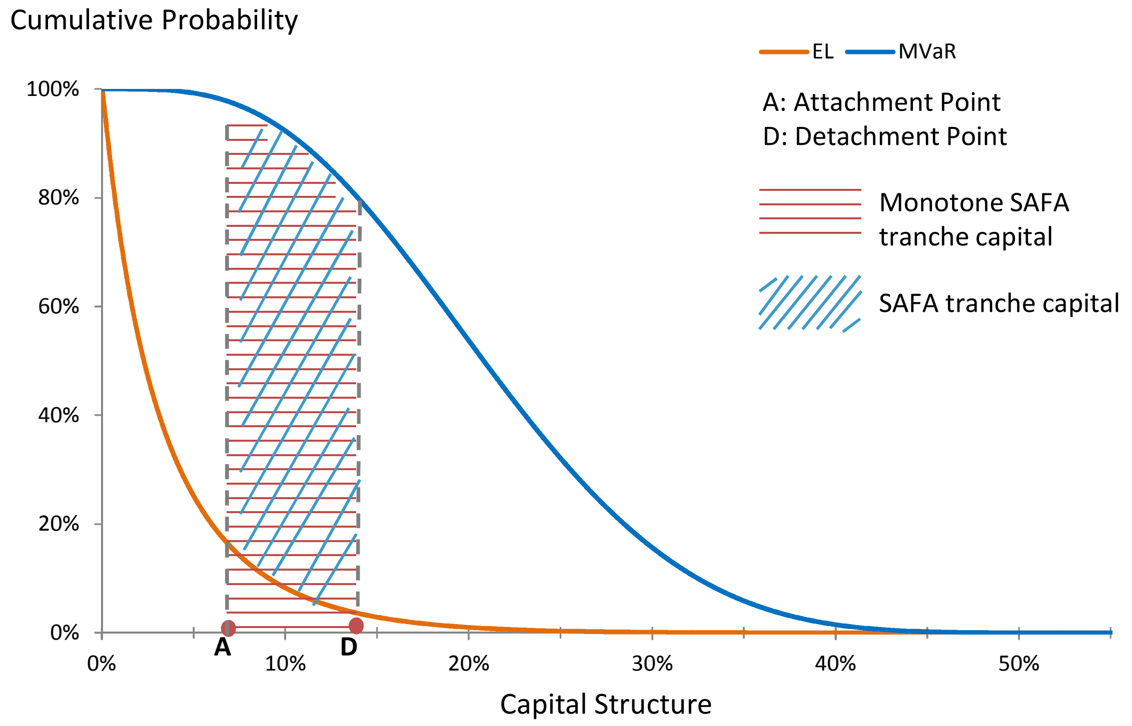


[30] In practise, it is not always possible to calculate exactly N_c, but it is fairly easy to approximate it. When relevant it is easier to show that N_c is greater than 100 than to calculate N_c exactly.

[31] See Basel (2006), paragraph 272. Normally, the Basel rules require values between 12% (low quality assets) and 24% (high quality assets). A reasonable "middle-ground" value would be: $\rho_{RW} = 18\%$.

[32] The diagram shows the capital when MRSF is set at 1.0. When it is greater than 1.0, such as a proposed 1.06 (for IRB banks) or 1.25 (for Standardised Banks), the area represent UL_{RW}, and a bandwidth representing a model risk charge (MRC) needs to be added.

Figure 2: Expected and Unexpected Losses



A graphical representation of the Simplified AFA is shown in Figure 1. The area between the Marginal VaR and the Expected Loss curves equals the Unexpected Loss. This area equals the Pool Risk Weights times 8%. The model may then be thought of as distributing this area over thin tranches according to their attachment point (i.e., their seniority).

Figure 2 exhibits the capital and Expected Losses for a discretely thick tranche and shows how the Monotone SAFA is constructed by adding to the Unexpected Loss the regulatory Expected Loss.

Finally, if required, one may further modify the SAFA to impose the monotonicity of capital charges.
If the regulatory expected loss (PD x LGD), including any related maturity adjustment, is included in the total amount of capital, both the AFA and the SAFA become monotonic. We refer to those versions as the Monotone AFA (MAFA) and Monotone SAFA (MSAFA).

Figures 1 and 2 illustrate the concepts just discussed. Figure 1 shows the total capital on all tranches (before any adjustment for monotonicity), equal to the pool risk weight multiplied by 8%, displayed as the area between the Marginal VaR and the Expected Loss curves. In effect, given the Risk Weight, the SAFA model attributes an amount of capital to the different tranches depending on the choice of the ρ^* parameter.

Figure 2 shows the SAFA-implied capital for a discretely thick tranche (with attachment and detachment points as shown in the figure) as the area between the MVaR and EL curves between *A* and *D*. The Monotone SAFA capital for the same tranche simply adds to the SAFA capital the area under the EL curve between *A* and *D*.

A key advantage of the Monotone SAFA is that there is no need to have a proxy ρ_{RW} for the systemic correlation, as the systemic correlation impacts only the distribution of the expected losses and not the distribution of the marginal value at risk.

In the monotone version, the capital of a tranche is reduced to the simpler definition:

$$Capital = EL_{Thick}(A, D, PD_{\alpha}, \rho^{*}, LGD) + MRC$$

Moreover, with a monotone version, there is no need to change the existing operational regulatory processes that currently exclude securitisation tranches when assessing whether margins on the bank's assets are sufficient to cover the regulatory expected loss of the bank.

SECTION 4 – ILLUSTRATIVE CALCULATIONS OF THE SAFA

This section presents illustrative calculations using different capital formulae, for 3 broad categories of assets using standardised risk weights. The three cases examined are a) large corporates with RWs of 100%, b) SMEs with RWs of 150% and c) residential mortgages with RWs of 35%. The detailed assumptions employed in the calculations are set out in Table 5.

All numbers are produced using a concentration correlation ρ^{*} of 10%, as this section is not concerned with the sensitivity to this parameter, but rather with (i) the sensitivity to p_{RW}, (ii) the impact of monotonicity, and (iii) the sensitivity to the premium parameter p of the SSFA.

Table 6 presents results for a securitisation with a pool of large corporate assets. We employ a standardised proxy for LGD_{RW} of 45% in this case. Table 7 shows results for a pool of SME corporate assets in which case we employ a conservative standardised proxy for LGD_{RW} of 75%. Table 8 presents results for a pool of RMBS assets for which we used a standardised proxy for LGD_{RW} of 25%.

For each category, we analyse how capital is affected by the inclusion of regulatory expected loss. We use values for p_{RW} of 8% and 20%. In the case of the SAFA, the proxy parameter p_{RW} creates regulatory expected losses below the Unexpected Loss distribution, and has the effect of reducing slightly the capital of junior tranches, distributing capital towards the junior mezzanines. Capital for the senior mezzanines is relatively insensitive to this parameter.

To illustrate this, in the case of the large corporate pool, increasing p_{RW} from 8% to 20% raises the capital for the junior mezzanines (Mezzanine 4 and Mezzanine 3) by 33% and 25% respectively, whereas the capital levels for the senior mezzanines (Mezzanine 2 and Mezzanine 1) increase by 3.9% and 0.1% only. For residential mortgage pools, the capital increases for the junior mezzanines (Mezzanine 4 and Mezzanine 3) are 39% and 19%, respectively, whereas for the increase for senior mezzanines (Mezzanine 2 and Mezzanine 1) have barely noticeable increases of 2.0% and 0.1%, respectively. One may note that, for the SME pool securitisation, the increase is distributed more evenly as there is a combined effect of a higher risk weight and a higher loss-given-default.

In the Monotone SAFA, the increase in capital mainly affects the junior tranche, however. The inclusion of the regulatory expected loss in the Monotone SAFA eliminates the phenomenon of decreasing capital when expected losses rise, that occurs under certain conditions. (Note that this phenomenon is already a feature of the current IRB formulae and, although counter-intuitive[33], it is managed in practice by requiring sufficient spread and/or provisions.)

In the case of the SME example, the increase in regulatory expected loss that occurs when p_{RW} increases from 8% to 20% reduces the capital allocated to the junior tranche from a RW of 946% to 860%, while increasing the capital allocated to mezzanine tranches. The Monotone SAFA instead increases the RW of the junior tranche from 1041% to 1094%, by converting this increased regulatory expected loss into capital requirement. The Monotone SAFA rules out situations in which the RW of a more junior tranche will be less than that of a more senior tranche regardless of thickness.

Note that by using a Model Risk Scaling Factor of 1.25 in the SAFA formula for Standardised Approach banks, the model risk charge creates a risk-sensitive floor of 20% of the underlying risk weight. This compares with a non-risk sensitive floor from the SSFA of 20% under the current Basel proposals. If 20% is the level of model risk that regulators regard as sensible for an average corporate portfolio risk weighted at 100%, it can be shown that 20% is too high for residential mortgages (it should be 7%), while it can be argued that it is too low for SMEs (it should be 30%). The problem arises because the floor in the SSFA is not risk sensitive.

[33] In IRB, when there is a large amount of regulatory EL in an asset, the UL cannot go higher than 1-EL.

Table 5: Assumptions for Capital Calculations

Large corporate pool, infinitely granular.							
	(A)	(B)	(C)	(D)	(E)	(F)	(G)
RW_Pool	100%	100%	100%	100%	100%	100%	100%
LGD_RW	45%	45%	45%	45%	-	-	-
Systemic Rho_RW	21%	21%	-	-	-	-	-
Concentration rho star	10%	10%	10%	10%	-	-	-
Model Risk Scaling Factor	1.25	1.25	1.25	1.25	-	-	-
p_RW	8%	20%	8%	20%	-	-	-
p (SSFA)	-	-	-	-	0.2	0.5	1.5
Model Risk Scaling Factor (SAFA and SSAFA)	1.25	1.25	1.25	1.25	-	-	-
SME corporate pool, infinitely granular.							
	(A)	(B)	(C)	(D)	(E)	(F)	(G)
RW_Pool	150%	150%	150%	150%	150%	150%	150%
LGD_RW	75%	75%	75%	75%	-	-	-
Systemic Rho_RW	10%	10%	-	-	-	-	-
Concentration rho star	10%	10%	10%	10%	-	-	-
Model Risk Scaling Factor	1.25	1.25	1.25	1.25	-	-	-
p_RW	8%	20%	8%	20%	-	-	-
p (SSFA)	-	-	-	-	0.2	0.5	-
Model Risk Scaling Factor (SAFA and SSAFA)	1.25	1.25	1.25	1.25	-	-	-
Residential mortgage pool, infinitely granular.							
RW_Pool	35%	35%	35%	35%	35%	35%	35%
LGD_RW	25%	25%	25%	25%	-	-	-
Systemic Rho_RW	15%	15%	-	-	-	-	-
Concentration rho star	10%	10%	10%	10%	-	-	-
Model Risk Scaling Factor	1.25	1.25	1.25	1.25	-	-	-
p_RW	8%	20%	8%	20%	-	-	-
p (SSFA)	-	-	-	-	0.3	0.5	1.5
Model Risk Scaling Factor (SAFA and SSAFA)	1.25	1.25	1.25	1.25	-	-	-

Table 6: Capital Calculations with Large Corporate Pool (100% RW)

	Approach:	SAFA	SAFA	Monotone SAFA	Monotone SAFA	SSFA	SSFA	SSFA
	Key Difference	**(pRW = 8%)**	**(pRW = 20%)**	**(pRW = 8%)**	**(pRW = 20%)**	**p=0.2**	**p=0.5**	**p=1.5**
		(A)	(B)	(C)	(D)	(E)	(F)	(G)
Thickness	Tranche				Capital			
70.0%	Senior	1.60%	1.60%	1.60%	1.60%	1.60%	1.60%	2.73%
5.0%	Mezzanine 1	1.60%	1.60%	1.60%	1.61%	1.60%	1.60%	19.83%
5.0%	Mezzanine 2	1.68%	1.75%	1.68%	1.77%	1.60%	2.84%	30.09%
5.0%	Mezzanine 3	2.64%	3.30%	2.65%	3.41%	1.60%	9.92%	45.64%
5.0%	Mezzanine 4	9.70%	12.94%	9.75%	13.43%	8.77%	34.62%	69.23%
10.0%	Junior	60.99%	59.00%	66.08%	71.49%	91.42%	95.74%	98.42%
100.0%	*Total Tranches After Securitisation*	*8.00%*	*8.00%*	*8.51%*	*9.28%*	*10.94%*	*13.14%*	*19.99%*
100.0%	*Total Pool Before Securitisation*	*8.00%*	*8.00%*	*8.00%*	*8.00%*	*8.00%*	*8.00%*	*8.00%*
	Ratio After / Before	1.00	1.00	1.06	1.16	1.37	1.64	2.50
	Floor (RW%)	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%
	Floor and underlying risk	Risk sensitive	Risk sensitive	Risk sensitive	Risk sensitive	Fixed	Fixed	Fixed
Thickness	Tranche				Risk Weights			
70.0%	Senior	20%	20%	20%	20%	20%	20%	34%
5.0%	Mezzanine 1	20%	20%	20%	20%	20%	20%	248%
5.0%	Mezzanine 2	21%	22%	21%	22%	20%	36%	376%
5.0%	Mezzanine 3	33%	41%	33%	43%	20%	124%	570%
5.0%	Mezzanine 4	121%	162%	122%	168%	110%	433%	865%
10.0%	Junior	762%	737%	826%	894%	1143%	1197%	1230%
100.0%	*Total Tranches After Securitisation*	*100%*	*100%*	*106%*	*116%*	*137%*	*164%*	*250%*
100.0%	*Total Pool Before Securitisation*	*100%*	*100%*	*100%*	*100%*	*100%*	*100%*	*100%*
	Ratio After / Before	1.00	1.00	1.06	1.16	1.37	1.64	2.50
Memorandum items								
RW Instability Ratio Mezzanine 2 / Mezzanine 1		*1.05*	*1.09*	*1.05*	*1.10*	*1.00*	*1.78*	*1.52*
RW Instability Ratio Mezzanine 3 / Mezzanine 2		*1.57*	*1.89*	*1.57*	*1.93*	*1.00*	*3.49*	*1.52*
RW Instability Ratio Mezzanine 4 / Mezzanine 3		*3.67*	*3.92*	*3.68*	*3.94*	*5.48*	*3.49*	*1.52*
RW Instability Ratio Junior / Mezzanine 4		*6.29*	*4.56*	*6.78*	*5.32*	*10.43*	*2.77*	*1.42*

Table 7: Capital Calculations with SME Corporate Pool (150% RW)

	Approach:	SAFA	SAFA	Monotone SAFA	Monotone SAFA	SSFA	SSFA	SSFA
		(pRW = 8%)	(pRW = 20%)	(pRW = 8%)	(pRW = 20%)	p=0.2	p=0.5	p=1.5
		(A)	(B)	(C)	(D)	(E)	(F)	(G)
Thickness	Tranche				Capital			
70.0%	Senior	2.42%	2.43%	2.42%	2.43%	1.60%	1.60%	9.27%
5.0%	Mezzanine 1	3.24%	3.85%	3.24%	3.86%	1.60%	7.77%	42.41%
5.0%	Mezzanine 2	5.68%	7.51%	5.69%	7.54%	1.60%	17.88%	55.98%
5.0%	Mezzanine 3	13.30%	17.56%	13.31%	17.72%	12.04%	41.15%	73.91%
5.0%	Mezzanine 4	32.62%	39.50%	32.67%	40.23%	74.25%	87.22%	95.27%
10.0%	Junior	75.67%	68.78%	83.31%	87.50%	100.00%	100.00%	100.00%
100.0%	*Total Tranches After Securitisation*	*12.00%*	*12.00%*	*12.77%*	*13.92%*	*15.59%*	*18.82%*	*29.86%*
100.0%	*Total Pool Before Securitisation*	*12.00%*	*12.00%*	*12.00%*	*12.00%*	*12.00%*	*12.00%*	*12.00%*
	Ratio After / Before	1.00	1.00	1.06	1.16	1.30	1.57	2.49
	Floor (RW%)	30.0%	30.0%	30.0%	30.0%	20.0%	20.0%	20.0%
	Floor and underlying risk	Risk sensitive	Risk sensitive	Risk sensitive	Risk sensitive	Fixed	Fixed	Fixed
Thickness	Tranche				Risk Weights			
70.0%	Senior	30%	30%	30%	30%	20%	20%	116%
5.0%	Mezzanine 1	41%	48%	41%	48%	20%	97%	530%
5.0%	Mezzanine 2	71%	94%	71%	94%	20%	224%	700%
5.0%	Mezzanine 3	166%	219%	166%	222%	150%	514%	924%
5.0%	Mezzanine 4	408%	494%	408%	503%	928%	1090%	1191%
10.0%	Junior	946%	860%	1041%	1094%	1250%	1250%	1250%
100.0%	*Total Tranches After Securitisation*	*150%*	*150%*	*160%*	*174%*	*195%*	*235%*	*373%*
100.0%	*Total Pool Before Securitisation*	*150%*	*150%*	*150%*	*150%*	*150%*	*150%*	*150%*
	Ratio After / Before	1.00	1.00	1.06	1.16	1.30	1.57	2.49
Memorandum items								
RW Instability Ratio Mezzanine 2 / Mezzanine 1		*1.75*	*1.95*	*1.75*	*1.95*	*1.00*	*2.30*	*1.32*
RW Instability Ratio Mezzanine 3 / Mezzanine 2		*2.34*	*2.34*	*2.34*	*2.35*	*7.52*	*2.30*	*1.32*
RW Instability Ratio Mezzanine 4 / Mezzanine 3		*2.45*	*2.25*	*2.45*	*2.27*	*6.17*	*2.12*	*1.29*
RW Instability Ratio Junior / Mezzanine 4		*2.32*	*1.74*	*2.55*	*2.17*	*1.35*	*1.15*	*1.05*

Table 8: Capital Calculations with Prime Residential Mortgage Pool (35% RW)

	Approach:	SAFA	SAFA	Monotone SAFA	Monotone SAFA	SSFA	SSFA	SSFA
		(pRW = 8%)	(pRW = 20%)	(pRW = 8%)	(pRW = 20%)	p=0.3	p=0.5	p=1.5
		(A)	(B)	(C)	(D)	(E)	(F)	(G)
Thickness	Tranche				Capital			
85.0%	Senior	0.56%	0.56%	0.56%	0.56%	1.60%	1.60%	1.60%
2.5%	Mezzanine 1	0.56%	0.56%	0.56%	0.56%	1.60%	1.60%	7.48%
2.5%	Mezzanine 2	0.57%	0.58%	0.57%	0.59%	1.60%	1.60%	13.57%
2.5%	Mezzanine 3	0.76%	0.90%	0.76%	0.92%	1.60%	1.62%	24.61%
2.5%	Mezzanine 4	2.80%	3.88%	2.81%	3.99%	2.32%	9.68%	44.63%
5.0%	Junior	44.13%	43.52%	47.71%	52.41%	71.58%	78.18%	90.25%
100.0%	*Total Tranches After Securitisation*	*2.80%*	*2.80%*	*2.98%*	*3.25%*	*5.12%*	*5.63%*	*8.13%*
100.0%	*Total Pool Before Securitisation*	*2.80%*	*2.80%*	*2.80%*	*2.80%*	*2.80%*	*2.80%*	*2.80%*
	Ratio After / Before	1.00	1.00	1.06	1.16	1.83	2.01	2.90
	Floor (RW%)	7.0%	7.0%	7.0%	7.0%	20.0%	20.0%	20.0%
	Floor and underlying risk	Risk sensitive	Risk sensitive	Risk sensitive	Risk sensitive	Fixed	Fixed	Fixed
Thickness	Tranche				Risk Weights			
85.0%	Senior	7%	7%	7%	7%	20%	20%	20%
2.5%	Mezzanine 1	7%	7%	7%	7%	20%	20%	94%
2.5%	Mezzanine 2	7%	7%	7%	7%	20%	20%	170%
2.5%	Mezzanine 3	10%	11%	10%	11%	20%	20%	308%
2.5%	Mezzanine 4	35%	48%	35%	50%	29%	121%	558%
5.0%	Junior	552%	544%	596%	655%	895%	977%	1128%
100.0%	*Total Tranches After Securitisation*	*35%*	*35%*	*37%*	*41%*	*64%*	*70%*	*102%*
100.0%	*Total Pool Before Securitisation*	*35%*	*35%*	*35%*	*35%*	*35%*	*35%*	*35%*
	Ratio After / Before	1.00	1.00	1.06	1.16	1.83	2.01	2.90
Memorandum items								
RW Instability Ratio Mezzanine 2 / Mezzanine 1		*1.02*	*1.04*	*1.02*	*1.05*	*1.00*	*1.00*	*1.81*
RW Instability Ratio Mezzanine 3 / Mezzanine 2		*1.33*	*1.54*	*1.33*	*1.56*	*1.00*	*1.01*	*1.81*
RW Instability Ratio Mezzanine 4 / Mezzanine 3		*3.68*	*4.30*	*3.68*	*4.35*	*1.45*	*5.96*	*1.81*
RW Instability Ratio Junior / Mezzanine 4		*15.78*	*11.22*	*17.01*	*13.14*	*30.80*	*8.07*	*2.02*

SECTION 5 – RELATION BETWEEN SSFA AND SIMPLIFIED AFA

In this section, we provide a reconciliation of the SSFA and AFA parameters, showing how reasonable values for the AFA parameters (about which one may adduce empirical experience) provide intuition for appropriate values for the SSFA calibration.

The capital for a thin tranche in the SSFA for an attachment point A, with $L = A - K_A$, is given by:

$$K_{SSFA}\ (L, L^{+}) \approx e^{a \cdot L} \tag{11}$$

Here, $a = \frac{-1}{p\, K_A}$. For the same attachment point, the AFA yields the following capital value:

$$K_{AFA}(A, A^{+}) \approx \mathrm{N}\left(\frac{N^{-1}\left(PD_{K_A}\right) - N^{-1}\left(\frac{\mathrm{A}}{LGD}\right)\sqrt{1-\rho^{*}}}{\sqrt{\rho^{*}}}\right) \tag{12}$$

For a given p, one may derive the ρ^{*} that yields the same capital value by solving the implicit equation below:

$$K_{AFA}(A, A^{+}) = K_{SSFA}(L, L^{+}) \tag{13}$$

After some manipulation, the solution reduces to solving a quadratic equation and hence the implied ρ^{*} value is available in closed form if a solution exists. The full mathematical development in given in the Appendix 5, but can be presented numerically below.

Tables 9 to 11 below show the results of such calculations for pool containing (i) investment grade large corporate loans, (ii) SME corporate loans and (iii) prime mortgage pools.

For investment grade corporate pools, it can be seen in Table 9 that a ρ^{*} of about 10%, corresponds approximately to a SSFA parameter value of $p = 0.2$. The SSFA market risk rule in the US is currently employs the value $p = 0.5$, and this would correspond to an implied ρ^{*} of 25% for high attachment points.

Note, however, that there is hardly any ρ^{*} parameter that could justify an SSFA parameter value of $p = 1.5$, as currently proposed in Basel (2012). The set of equations is not numerically solvable, or in other words setting $p = 1.5$ cannot be approximated in a standard two factor model like the Pykhtin-Dev/AFA model.

As one may observe from Table 10, for SME corporate pools, ρ^{*} parameter of about 10% corresponds to a SSFA parameter value of $p = 0.2$. The SSFA market risk rule in the US, currently using $p = 0.5$, would correspond to an implied ρ^{*} of 35% for high attachment points. Setting $p = 1.5$ as in the proposed Basel (2012), the set of equations is not numerically solvable, i.e., the SSFA is incompatible with a two-factor model.

For RMBS (see Table 11), the level of ρ^{*} of about 3%, which corresponds to this asset class[34], corresponds to an SSFA parameter value of $p = 0.1$. Such a value is of the order of the magnitudes proposed by Pykthin and Mingo when the original SSFA was developed.

[34] See page 18, Duponcheele et al (2013)

Table 9: Corporate Pool (Standardised Approach) with 100% risk weights and 45% regulatory LGD

Inputs		LARGE CORPORATE POOL														
RW	100%	The AFA ρ^* parameter implied by different values of the SSFA parameter p														
LGD_{RW}	45%	p														
Attachment Point		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
	30%	5%	10%	16%	22%	29%	37%	45%	54%	63%	72%	81%	90%	96%	100%	NS
	25%	4%	9%	15%	21%	29%	39%	49%	62%	75%	90%	NS	NS	NS	NS	NS
	20%	4%	8%	15%	23%	34%	49%	70%	NS	NS	NS	NS	NS	NS	NS	NS
A	15%	3%	9%	18%	36%	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS
	12%	3%	12%	54%	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS
	8%	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$
	0%	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$

Table 10: SME Corporate Pool (Standardised Approach)

Inputs		SME CORPORATE POOL														
RW	150%	The AFA ρ^* parameter implied by different values of the SSFA parameter p														
LGD_{RW}	75%	p														
Attachment Point		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
	30%	3%	7%	13%	21%	32%	47%	71%	NS	NS	NS	NS	NS	NS	NS	NS
	25%	3%	8%	15%	26%	47%	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS
	20%	3%	9%	23%	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS
A	15%	5%	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS
	12%	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$
	8%	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$
	0%	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$

Table 11: Prime Residential Mortgage Pool (Standardised Approach)

Inputs		HIGH QUALITY RESIDENTIAL MORTGAGE POOL														
RW	35%	The AFA ρ^* parameter implied by different values of the SSFA parameter p														
LGD_{RW}	25%	p														
Attachment Point		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
	15.0%	3%	7%	10%	15%	19%	24%	29%	35%	41%	47%	53%	60%	67%	75%	82%
	12.5%	3%	6%	10%	14%	19%	24%	30%	37%	44%	52%	61%	71%	83%	95%	NS
	10.0%	3%	6%	10%	14%	20%	26%	34%	43%	55%	71%	NS	NS	NS	NS	NS
A	7.5%	2%	6%	10%	16%	24%	36%	55%	NS	NS	NS	NS	NS	NS	NS	NS
	5.0%	2%	7%	15%	43%	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS
	2.8%	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$	$=K_A$
	0%	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$	$<K_A$

A ρ^* value of about 10% corresponds to a value of $p = 0.3$. The SSFA market risk rule in the US, currently using $p = 0.5$, would correspond to an implied ρ^* of 21% for high attachment points. But setting $p = 1.5$ as in the proposed Basel (2012), the set of equations is incompatible with a two factor model and is not numerically solvable.

SECTION 6 – CONCLUSION

In a recent speech[35], Wayne Byres set out the principles that should guide the design of regulatory policies, stating that such policies should be:

1. Comprehensive, yet simple;
2. Strong, but not burdensome;
3. Risk-based, yet easy to understand and compare;
4. Flexible and adaptable, yet consistently applied;
5. Suitable for normal times, but founded on the lessons from crises;
6. Built on consensus, but also on the broadest possible engagement; and
7. Utilising appropriately the relative strengths of both regulation (rules) and supervision (oversight).

In our view, the current proposals for securitisation capital contained in BCBS (2012) do not adhere to these broad principles. The SSFA has some desirable characteristics, specifically transparency and simplicity for investors, originators and dealers, but fails in other regards. The MSFA is deficient in most of the dimensions identified by Byres.

Following feedback from the industry, there has been extensive discussion of several aspects of the proposed securitisation capital framework (for example on calibration, simplicity of use, risk-sensitivity) but so far there has been somewhat less focus on Byres' point 4, i.e., on consistency. In particular, the lack of consistency between the MSFA and the SSFA is a significant weakness of the current proposals since it implies sharp discontinuities in capital levels depending on what information is available to a bank.

In this paper, we have developed a simplified version of the Arbitrage Free Approach proposed in Duponcheele et al (2013). Because the so-called Simplified AFA is coherent in underlying assumptions with the AFA itself, the two approaches combined offer a consistent set of approaches that may be used by investor or issuer banks.

Both the AFA and the SAFA, with or without monotonicity, are easily adaptable to real-life securitisation situations, with or without granularity, with or without heterogeneity, with or without mixed pools, with SA and/or IRB data. In its simplest form, the Monotone SAFA could be constructed with just one user-supplied input: the risk weight RW of the underlying asset pool.

The resulting AFA and SAFA approaches are comprehensive yet simple, conservative but not burdensome, risk-based yet easy to understand and compare, flexible yet consistently applied, suitable for normal times but built on lessons from the crisis, built on consensus with a broad engagement but leaving the regulators in control.

[35] "Simplicity, Risk sensitivity and Comparability: the Regulatory Balancing Act," Wayne Byres, 25-26 February 2013.

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Table 12: Comparison of Approaches with PD, LGD Inputs

Method (PD, LGD, rho)	ASRF	Concentration Factor	MVaR Statistical Measure for Expected Loss when Bank under stress at 99.9%	Regulatory Expected Loss Treatment	Source of distribution	Risk Sensitive Model Risk Charge	Maturity sensitive
IRBA	Yes	N/A	Yes	Excluded	Vasicek	Yes, 6% times UL	Yes for size
Original SSFA	Yes	Implicit	Yes, implicit but premium added	Included	Exponential (capital)	No, fixed floor	Yes for size
SFA	Yes	No	Yes	Included	Random Tranches (tau), granularity (delta), smoothing (omega)	No, fixed floor	Yes for size
MSFA	Yes	No	No, Expected Shortfall at 99.7%	Included and multiplied several times over	Maturity modelling and volatility, Recovery variance (tau), smoothing (omega)	No, fixed floor	Yes for size and dispersion
AFA	Yes	Yes	Yes	Excluded	Vasicek, Concentration (rho star), granularity (delta)	Yes, 6% times UL	Yes for size and dispersion(+)
Monotonic AFA	Yes	Yes	Yes	Included	Vasicek, Concentration (rho star), granularity (delta)	Yes, 6% times UL	Yes for size and dispersion(+)
SAFA	Yes	Yes	Yes	Excluded, by proxy	Vasicek, Concentration (rho star), granularity (delta)	Yes, 6% times UL	Yes for size and dispersion(+)
Monotonic SAFA	Yes	Yes	Yes	Included, by proxy	Vasicek, Concentration (rho star), granularity (delta)	Yes, 6% times UL	Yes for size and dispersion(+)

Table 13: Comparison of Approaches with RW Inputs

Method (RW)	ASRF	Concentration Factor	Statistical Measure for Expected Loss when Bank under stress at 99.9%	Regulatory Expected Loss Treatment	Source of distribution	Risk Sensitive Model Risk Charge	Maturity sensitive
SA	Implicit	N/A	Yes, implicit	Excluded	N/A	N/A	No for size
SSFA	Implicit	Implicit	Yes, implicit but premium added	Excluded	Exponential (capital)	No, fixed floor	No for size
SAFA	Yes	Yes	Yes	Excluded, by proxy	Vasicek, concentration (rho star), granularity (delta)	Yes, 25% times UL	No for size, Only for dispersion(+)
Monotonic SAFA	Yes	Yes	Yes	Included, by proxy	Vasicek, concentration (rho star), granularity (delta)	Yes, 25% times UL	No for size, Only for dispersion(+)

(+): a separate paper on the maturity effect on the dispersion of the distribution is currently being drafted by the authors.

APPENDIX

A1: STEP-BY-STEP IMPLEMENTATION OF MONOTONE SAFA

This 'monotone'-simplified version does not require a proxy for the systemic correlation to determine the expected loss contribution, but uses a regulatory multiplier for the expected loss to ensure that the unexpected losses are appropriately distributed.

With the following inputs:

- the risk weight of the pool, RW_{Pool} (including of any scaling factor)
- the regulatory estimate of loss-given default LGD_{RW} set by regulators for the asset class (see tables);
- the new 'stressed' correlation ρ^*, set by regulators for the asset class;
- the multiplier to include regulatory expected loss, $M_{EL} = 1 + p_{RW}$
- the model risk scaling factor $MRSF$ (1.25 for SA and 1.06 for IRBA)
- for the tranche T, given its effective attachment point A, and effective detachment point D

Calculate:

$$UL_{Pool} = \frac{RW_{Pool}}{12.5 \times MRSF} \quad \text{(A1.1)}$$

$$LGD_{Pool} = LGD_{RW} \quad \text{(A1.2)}$$

[36]

$$SPD'_{Pool} = \frac{UL_{Pool} \times M_{EL}}{LGD_{Pool}} \quad \text{(A1.3)}$$

$$s\rho = \rho^* \quad \text{(A1.4)}$$

$$SPD_T(A) = VasicekPD_T(A, SPD'_{Pool}, LGD_{Pool}, s\rho) \quad \text{(A1.5)}$$

$$SPD_T(D) = VasicekPD_T(D, SPD'_{Pool}, LGD_{Pool}, s\rho) \quad \text{(A1.6)}$$

$$SLGD_T(A, D) = VasicekLGD_T(A, D, SPD_T(A), SPD_T(D), SPD'_{Pool}, LGD_{Pool}, s\rho) \quad \text{(A1.7)}$$

$$\%CR_{RW_T} = SPD_T(A) \cdot SLGD_T(A, D) + ((MRSF - 1) \cdot UL_{Pool}) \quad \text{(A1.8)}$$

[37]

$$\$CR_{RW_T} = (\$D - \$A) \cdot \%CR_{RW_T} \quad \text{(A1.9)}$$

Final coherence check (prior to adjustments): $\sum_T \$CR_{RW_T}$ should be equal to $\frac{1}{12.5} \times RW_{Pool}$. The VBA function $VasicekPD_T$ and $VasicekLGD_T$ are given at the end of this paper.

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[36] This part can be changed with a weighted average pool LGD such that $LGD_{Pool} = \frac{\sum_i (LGD_{RW,i} \cdot EAD_i)}{\sum_i EAD_i}$.

[37] Because of the non-deduction of EL and the addition of the MRC, for the most junior and ultra thin tranches, equation A1.8 can go over 1250%. This numerical issue disappears with the thickness of real tranches. A numerical control could be added to ensure that no tranches go over 1250% risk weight.

A2: Monotone SAFA with Granularity Adjustment

This variant takes the granularity effect and its impact on the distribution of unexpected loss contribution. This is particularly relevant for those non-retail securitisations (typically non-granular CMBS)

For exposures under the wholesale framework, the granularity adjustment would be calculated as in paragraph 633 of BCBS (2006), with N_c being the effective number of consolidated exposures. In this case:

$$\delta_{RW} = \frac{1}{N_c} \qquad where \qquad N_c = \frac{\left(\sum_{i,i\subset c} EAD_i\right)^2}{\left(\sum_{i,i\subset c} EAD_i^2\right)} \tag{A2.1}$$

In that case the implementation is like in Appendix 1-A but replacing the stressed correlation by:

$$s\rho = \rho^* + \delta_{RW}\,(1-\rho^*) \tag{A2.2}$$

Please note that for securitisations of retail or SME underlyings, granularity is very high, and there is no point in attempting a granularity adjustment in the Vasicek distribution. For retail exposures: $\delta_{RW} = 0\%$.

A3: Step-by-Step Implementation of the SAFA

This simplified version takes into account the expected loss contribution and the granularity effect and its impact on the distribution of unexpected loss contribution. In contrast to the Monotonic Simplified AFA, it then removes the expected loss contribution from the capital of the securitisation tranches.

With the following general inputs:

- the systemic correlation ρ_{RW}, set by regulators for the asset class (see tables)
- p_{RW} a regulatory estimate of expected loss as a proportion of unexpected loss ($see\ tables$)
- the new 'stressed' correlation ρ^*, set by regulators for the asset class
- the model risk scaling factor $MRSF$ (e.g. 1.25 for SA and 1.06 for IRBA)

With the following parameters for each asset $\boldsymbol{i}$:

1. the risk weight RW_i (including of any scaling factor)[38]
2. the exposure at default EAD_i
3. the regulatory estimate of loss-given default $LGD_{RW,i}$ set by regulators for the asset $\boldsymbol{i}$ (see tables);

Calculate for the asset, the asset dependent value:

- the implied unexpected loss of the asset:

$$UL_{RW,i} = \frac{RW_i}{12.5 \times MRSF} \tag{A3.1}$$

[38] In IRBA, this would be $RW_i = 12.5 \times K \times 1.06$

Calculate pool parameters:

- the weighted average implied unexpected loss of the pool:

$$UL_{Pool} = \frac{\sum_i (UL_{RW,i} \cdot EAD_i)}{\sum_i EAD_i} \tag{A3.2}$$

- the implied expected loss of the pool:

$$EL_{Pool} = p_{RW} \cdot UL_{Pool} \tag{A3.3}$$

- the implied loss given default of the pool:

$$LGD_{Pool} = \frac{\sum_i (LGD_{RW,i} \cdot EAD_i)}{\sum_i EAD_i} \tag{A3.4}$$

- the implied probability of default of the pool:

$$PD'_{Pool} = \frac{EL_{Pool}}{LGD_{Pool}} \tag{A3.5}$$

- the implied stressed probability of default of the pool:

$$SPD'_{Pool} = \frac{UL_{Pool} + EL_{Pool}}{LGD_{Pool}} \tag{A3.6}$$

- the effective number of consolidated exposures in the pool, if relevant:

$$N_c = \frac{\left(\sum_{i,i \subset c} EAD_i\right)^2}{\left(\sum_{i,i \subset c} EAD_i^2\right)} \tag{A3.7}$$

- the granularity adjustment:

$$\delta_{RW} = \frac{1}{N_c} \tag{A3.8}$$

Coherence check: the unexpected loss contribution to Bank of the pool should be equal to:

$$\%K_{RW,Pool} = SPD'_{Pool} \cdot LGD_{Pool} - PD'_{Pool} \cdot LGD_{Pool} = UL_{Pool} \tag{A3.9}$$

Calculate for each tranche, the tranche-dependent and pool-dependent values:

For the tranche T with the effective attachment point of the tranche A, and the effective detachment point of the tranche D, the contribution of the $Pool$ to the tranche need the following intermediary steps:

For the Expected Loss Component:

- the pool correlation ρ_{Pool}, is given by:

$$\rho_{Pool} = \rho_{RW} + (1 - \rho_{RW}) \cdot \rho^* \tag{A3.10}$$

- the Vasicek granularity adjusted pool correlation:

$$\rho'_{Pool} = \rho_{Pool} + \delta_{RW} \cdot (1 - \rho_{Pool}) \tag{A3.11}$$

- the probability of default of the tranche boundaries $PD_T(A)$ and $PD_T(D)$:

$$PD_T(A) = VasicekPD_T(A, PD'_{Pool}, LGD_{Pool}, \rho'_{Pool}) \quad \text{(A3.12)}$$

$$PD_T(D) = VasicekPD_T(D, PD'_{Pool}, LGD_{Pool}, \rho'_{Pool}) \quad \text{(A3.13)}$$

- the loss given default of the tranche $LGD_T(A, D)$:

$$LGD_T(A, D) = VasicekLGD_T(A, D, PD_T(A), PD_T(D), PD'_{Pool}, LGD_{Pool}, \rho'_{Pool}) \quad \text{(A3.14)}$$

- the Marginal Contribution of the tranche T to the Expected Loss of the Bank:

$$\%MC_T EL_{Bank} = PD_T \cdot LGD_T \quad \text{(A3.15)}$$

For the Stressed Expected Loss Component:

- the Vasicek granularity adjusted stressed correlation:

$$s\rho'_{Pool} = \rho^* + \delta_{RW} \cdot (1 - \rho^*) \quad \text{(A3.16)}$$

- the stressed PD of the tranche boundaries $SPD_T(A)$ and $SPD_T(D)$:

$$SPD_T(A) = VasicekPD_T(A, SPD'_{Pool}, LGD_{Pool}, s\rho'_{Pool}) \quad \text{(A3.17)}$$

$$SPD_T(D) = VasicekPD_T(D, SPD'_{Pool}, LGD_{Pool}, s\rho'_{Pool}) \quad \text{(A3.18)}$$

- the Stressed LGD of the tranche $SLGD_T(A, D)$:

$$SLGD_T(A, D) = VasicekLGD_T(A, D, SPD_T(A), SPD_T(D), SPD'_{Pool}, LGD_{Pool}, s\rho'_{Pool}) \quad \text{(A3.19)}$$

- the Marginal Contribution of the tranche T to the Value at Risk of the Bank, at the financial stability confidence level (*FSCL*=99.9%):

$$\%MC_T VaR_{Bank,FSCL} = SPD_T(A) \cdot SLGD_T(A, D) \quad \text{(A3.20)}$$

For the Model Risk Charge Component:

- the Marginal Contribution of the tranche T to the Model Risk Charge of the Bank:

$$\%MC_T MRC_{Bank} = \left((MRSF - 1) \cdot \%K_{RW,Pool}\right) \quad \text{(A3.21)}$$

Apply the Basel II formula for the 3 Unexpected Loss Components:

$$\%CR_{RW_T} = \%MC_T VaR_{Bank,FSCL} - \%MC_T EL_{Bank} + \%MC_T MRC_{Bank} \quad \text{(A3.22)}$$

Multiply by the thickness to move from percentage notation (applied to the tranche's notional) to 'dollar' notation (applied to the pool's notional):

$$\$CR_{RW_T} = (\$D_T - \$A_T) \cdot \%CR_{RW_T} \tag{A3.23}$$

$$\$K_{RW,Pool} = \$1 \cdot \%K_{RW,Pool} \tag{A3.24}$$

Final coherence check (prior to adjustments): $\sum_T \$CR_{RW_T}$ should be equal to $\$K_{RW,Pool}$.

The VBA function $VasicekPD_T$ and $VasicekLGD_T$ are given at the end of this paper.

A4: BASIC VBA FUNCTIONS

The function $VasicekPD_T(A, p, lgd, \rho)$ and $VasicekLGD_T(A, D, p_T(A), p_T(D), p, lgd, \rho)$ are given below:

$$VasicekPD_T(A, p, lgd, \rho) = \begin{cases} if\ A \geq lgd,\ then\ p_T(A) = 0\% \\ if\ 0 < A < lgd,\ then \qquad p_T(A) = N\left(\frac{N^{-1}(p) - \sqrt{1-\rho} \cdot N^{-1}\left(\frac{A}{lgd}\right)}{\sqrt{\rho}}\right) \\ if\ A = 0,\ then\ p_T(A) = 100\% \end{cases} \tag{A4.1}$$

$$VasicekLGD_T(A, D, p_T(A), p_T(D), p, lgd, \rho) \\ = \begin{cases} if\ A \geq lgd,\ then\ lgd_T = 0\% \\ if\ 0 \leq A < lgd,\ then \qquad lgd_T = \frac{\frac{p_T(D)}{p_T(A)} \cdot D - A}{D - A} + \frac{lgd}{(D - A)} \cdot \left(\frac{BV(p, p_T(A), \rho) - BV(p, p_T(D), \rho)}{p_T(A)}\right) \end{cases}$$

where $BV(p, p_T(X), \rho)$

$$= \begin{cases} if\ X \geq lgd, then \qquad BV(p, p_T(X), \rho) = 0\% \\ if\ 0 < X < lgd, then \quad BV(p, p_T(X), \rho) = N_2\left(N^{-1}(p), N^{-1}\left(p_T(X)\right), \sqrt{\rho}\right) \\ if\ X = 0,\ then \qquad BV(p, p_T(X), \rho) = p \end{cases} \tag{A4.2}$$

with $N_2(x, y, r)$ being the bivariate cumulative standard normal distribution function[39].

[39] The $N_2(\ \ )$ function is easily implementable in Excel using VBA.

A5: THE LINK BETWEEN p OF THE SSFA AND ρ^* OF THE AFA

The SSFA capital charge K_{SSFA} for a given tranche is determined by the following formula:

$$K_{SSFA}(L,U) = \frac{\left(e^{a\,U} - e^{a\,L}\right)}{a\,(U-L)} \tag{A5.1}$$

Here:

$$L = A - K_A$$

$$U = D - K_A$$

$$a = \frac{-1}{p\,K_A}$$

$$K_A = (1-W)\,K_g + \frac{W}{2} \tag{A5.2}$$

K_g is the weighted average Basel risk-based capital requirement of the underlying portfolio. For most assets, it is 8% (i.e. with a risk-weight RW=100%). W is the percentage of the underlying portfolio that is currently defaulted or in serious delinquency. A is the tranche attachment point, D is the detachment point. In the US rules, p is 0.5 for securitisations and 1.5 for re-securitisations. In Basel (2012), the value p is 1.5 for securitisations.

Capital under the SSFA for a very thin Tranche is given by the following equation.

$$K_{SSFA}(L,L^+) \approx e^{a \cdot L} \tag{A5.3}$$

Under the AFA, capital for a similar thin tranche is as follows.

$$K_{AFA}(A,A^+) \approx \mathrm{N}\left(\frac{N^{-1}\left(PD'_{K_A}\right) - N^{-1}\left(\frac{\mathrm{A}}{LGD}\right)\sqrt{1-\rho^*}}{\sqrt{\rho^*}}\right) \tag{A5.4}$$

where $PD'_{K_A} = \frac{K_A}{LGD \times MRSF}$.

The implied AFA correlation for thin tranches may be derived as follows, with $L = A - K_A$:

$$K_{AFA}(A,A^+) = K_{SSFA}(L,L^+) \approx \mathrm{N}\left(\frac{N^{-1}\left(PD'_{K_A}\right) - N^{-1}\left(\frac{\mathrm{A}}{LGD}\right)\sqrt{1-\rho^*}}{\sqrt{\rho^*}}\right) = e^{a \cdot L} \tag{A5.5}$$

$$N^{-1}\left(PD'_{K_A}\right) - N^{-1}\left(\frac{\mathrm{A}}{LGD}\right)\sqrt{1-\rho^*} = N^{-1}\left(e^{a \cdot L}\right)\sqrt{\rho^*} \tag{A5.6}$$

$$\alpha\rho^* + \beta\sqrt{\rho^*} + \gamma = 0 \tag{A5.7}$$

where

$$\alpha = N^{-1}\left(\frac{\mathrm{A}}{LGD}\right)^2 + N^{-1}\left(e^{a \cdot L}\right)^2$$

$$\beta = -2 \cdot N^{-1}\left(e^{a \cdot L}\right) \cdot N^{-1}\left(PD'_{K_A}\right)$$

$$\gamma = N^{-1}\left(PD'_{K_A}\right)^2 - N^{-1}\left(\frac{\mathrm{A}}{LGD}\right)^2$$

This equation has a real solution if and only if the discriminant Δ is positive, i.e., if:

$$\Delta = \beta^2 - 4\alpha\gamma = 4N^{-1}\left(\frac{\mathrm{A}}{LGD}\right)^2\left(N^{-1}\left(\frac{\mathrm{A}}{LGD}\right)^2 + N^{-1}(e^{a\cdot L})^2 - N^{-1}\left(PD'_{K_A}\right)^2\right) > 0 \qquad \text{(A5.8)}$$

$$\begin{cases} \Delta \geq 0 \; and \; \sqrt{\Delta} \geq \beta \; and \; \frac{\mathrm{A}}{LGD} \geq 0.5 & then \quad \sqrt{\rho^*} = \frac{-\beta+\sqrt{\Delta}}{2\alpha} \\ \Delta \geq 0 \; and \; \sqrt{\Delta} \leq -\beta \; and \; \frac{\mathrm{A}}{LGD} < 0.5 & then \quad \sqrt{\rho^*} = \frac{-\beta-\sqrt{\Delta}}{2\alpha} \end{cases} \qquad \text{(A5.9)}$$

The numerical results presented in Table 9, 10 and 11 were produced using the formula in (A5.9) and using $MRSF = 1.25$.