How to Analyse Risk in Securitisation Portfolios:
A Case Study of European SME-loan-backed deals[1]

Executive summary

Returns on securitisation tranches depend on the performance of the pool of assets against which the tranches are secured. The non-linear nature of the dependence can create the appearance of regime changes in securitisation return distributions as tranches move more or less "into the money".

Reliable risk management requires an approach that allows for this non-linearity by modelling tranche returns in a 'look through' manner. This involves modelling risk in the underlying loan pool and then tracing through the implications for the value of the securitisation tranches that sit on top.

This note describes a rigorous method for calculating risk in securitisation portfolios using such a look through approach. Pool performance is simulated using Monte Carlo techniques. Cash payments are channelled to different tranches based on equations describing the cash flow waterfall. Tranches are re-priced using statistical pricing functions calibrated through a prior Monte Carlo exercise.

The approach is implemented within *Risk Controller™*, a multi-asset-class portfolio model. The framework permits the user to analyse risk return trade-offs and generate portfolio-level risk measures (such as Value at Risk (VaR), Expected Shortfall (ES), portfolio volatility and Sharpe ratios), and exposure-level measures (including marginal VaR, marginal ES and position-specific volatilities and Sharpe ratios).

We implement the approach for a portfolio of Spanish and Portuguese SME exposures. Before the crisis, SME securitisations comprised the second most important sector of the European market (second only to residential mortgage backed securitisations). In the current low interest environment, SME securitisations offer attractive returns. But, for investors to be confident in developing a portfolio of SME securitisations, fully satisfactory approaches to measuring and managing risk must be implemented.

Our portfolio of Spanish and Portuguese SME exposures has a 99.9% VaR of 8.20%, close to the traditional Basel I capital ratio of 8% (which is often associated with high BB-rated loans). We show how the tranche-level Marginal VaRs implied by our approach are strongly positively correlated with a low attachment point, a high Weighted Average Life (WAL) and low rating grades.

To benchmark the capital measures supplied by our Monte Carlo model, we also calculate MVaRs using a simple, stylised capital model, the Arbitrage Free Approach (AFA), introduced by Duponcheele, Perraudin and Totouom-Tangho (2013). We show that the numerical Monte Carlo model MVaRs and those implied by the AFA are closely correlated. (A regression of the numerical MVaRs on the AFA MVaRs yields an R-squared of 88%.)

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[1] This note was prepared by Jozsef Kutas and William Perraudin.

As another benchmarking exercise, we calculate the MVaRs implied by the SEC-IRBA and SEC-SA regulatory capital charges specified in BCBS (2014). Again, we show they are correlated with the numerical Monte Carlo MVaRs. (A regression of numerical MVaRs on SEC-IRBA and SEC-SA MVaRs yields an R^2 of 87%.)[2]

Taken together, the numerical and theoretical models implemented in this note provide a convincing analysis of the risk involved in holding a portfolio of securitisation tranches. As such, they permit investors to profit from the relatively high returns offered by securitisation portfolios while maintaining a cautious and prudent approach to risk.

1.   Introduction

Securitisation exposures depend in a non-linear fashion on the performance of the asset pools (typically comprising loans) against which they are secured. This non-linear dependence means that return volatility may change significantly over time as the effective level of subordination changes. Apparent regime changes in the riskiness of returns are simply the result of the non-linear dependence of tranche values on underlying pools.

Effective and robust risk management of securitisation portfolios should take account of such non-linearity. This note explains how to analyse risk in a securitisation portfolio using a 'look through' approach. This approach involves modelling the cash flow waterfall of securitisation deals constructed on top of a stochastic model of pool asset performance. The model we describe is implemented using Monte Carlo methods and can represent realistically complex cash flow waterfalls in a rigorous fashion.

We implement this Monte Carlo approach within a flexible portfolio modelling software called *Risk Controller*[TM]. The software supports analysis of multi-currency portfolios comprising bonds, equities and derivatives of various types. Hence, the contribution of securitisation exposures to wider portfolios of instruments may be accurately computed.

To illustrate the approach, we analyse a portfolio of Spanish and Portuguese Small and Medium Enterprise (SME) deals. We calculate portfolio risk statistics such as Value at Risk (VaR) and Expected Shortfall (ES) and marginal VaRs (denoted MVaRs) for individual securitisation exposures. We study features of the securitisation exposure that increase MVaRs for individual tranches. We find that MVaRs are strongly positively associated with low attachment points, long Weighted Average Life (WAL) and low ratings.

We compare the marginal VaRs generated using the Monte Carlo model with those implied by a dynamic version of the Arbitrage Free Approach (AFA) developed in Duponcheele, Perraudin and Totouom-Tangho (2013). This latter model is a stylised but rigorous model for calculating the capital for individual securitisation exposures. It is employed by the authors to shed light on appropriate levels of regulatory capital. The AFA model does not capture the full complexity of a cash-flow waterfall since it assumes no coupon or principal payments except at the final maturity of the securitisation. We find that the MVaRs obtained using the Monte Carlo model are highly correlated with those implied by the AFA. (When the former are regressed on the latter for our sample, the R-squared statistic is 88%.)

We compare the marginal VaRs generated using the Monte Carlo model with those implied by a dynamic version of the Arbitrage Free Approach (AFA) developed in Duponcheele, Perraudin and Totouom-Tangho (2013). This latter model is a stylised but rigorous model for calculating the marginal capital for securitisation exposures formulated to shed light on appropriate levels of regulatory capital. The model does not capture the full complexity of a cash-flow waterfall since it assumes no coupon or principal payments except at the final maturity of the securitisation. It also models defaults in pool loans within a one-period model rather than assuming multiple time periods.

We also compare numerically obtained Monte Carlo MVaRs with capital figures implied by regulatory formulae contained in BCBS (2014), namely the SEC-IRBA and SEC-SA approaches. These have been calibrated by the authorities using an ad hoc formula (the Simplified Supervisory Formula Approach (SSFA)) as an approximation to a model reportedly similar to the AFA. Again, we find high associations with R-squared statistics for a regression of the Monte Carlo MVaRs on the regulatory capital calculations of 88%.

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[2] While the analytical models yield MVaRs closely correlated with the numerical MVaRs for a specialised portfolio consisting of tranches in a single market (in this case, SME-backed deals in Spain and Portugal), note that they are not capable of robust generalisation to a multi-factor portfolio that includes tranches from different geographical regions and pools comprising other loan types. Also, within a narrower range of credit quality (for example tranches in our sample with MVaRs below the median), the correlation is lower.

These comparisons underline the robust nature of the numerical calculations involved in the Monte Carlo MVaR calculations. The AFA and regulatory formulae approaches presume that a single risk factor drives the bank balance sheet while another drives each individual securitisation pool. Such assumptions are not appropriate in the context of analysing risk in a portfolio of securitisations from different geographical regions and pool asset classes. The exercises reported here involve a portfolio limited to a single pool asset class (SME loans) and all originated in a single geographical region (Spain and Portugal). In this case, the number of underlying risk factors is small and one may regard the Monte Carlo and analytical approaches (the latter or which assumes a single common risk factor) as comparable.

Note that even when the Monte Carlo model is applied assuming few risk factors, it yields results that differ somewhat from the AFA (and the regulatory capital models) because in the Monte Carlo model a more realistic representation of cash flow is allowed for. To investigate how this affects the results, we regress the differences in MVaRs from the numerical model and the AFA on a set of capital drivers and find that for higher deal duration and attachment points, the numerical MVaRs are lower than the AFA-implied MVaRs. This may reflect the fact that in actual deals (as described more accurately in the numerical model) excess spread accumulates and protects senior tranches against defaults.

The structure of this note is as follows. Section 2 describes the securitisation portfolio we examine, setting out the assumptions adopted in calibrating the model. Sections 3 and 4 set out the methodology employed. Section 5 presents the results of our analysis. We first look at the performance of the securitisations in isolation, and then examine how the marginal VaRs of the securitisation exposures vary according to various exposure characteristics. Section 6 summarises the AFA capital model for securitisation tranches, and compares the risk statistics it implies with those supplied by the Monte Carlo model. It also compares results with regulatory capital calculations. Section 7 concludes.

2.   Description of the securitisation portfolio

This paper describes a Monte Carlo approach to modelling capital on securitisation portfolios and then applies it to a portfolio of Spanish and Portuguese securitisations. The portfolio employed in the illustrative calculations consists of 72 tranches from 25 SME-loan backed securitisations. 24 of the securitisations are Spanish, and 1 is Portuguese. The total value of the tranches is EUR 4.255 billion, and the total amount held in reserve is EUR 650 million.

Figure 1 shows the breakdown of the exposures by rating. As one may observe, the portfolio contains very few AAA-rated tranches. Most tranches have ratings of A or BBB but there are substantial numbers of exposures rated BB, B, CCC and even default. Interpretation of ratings is, in this case, complicated by the fact that the ratings agencies apply sovereign rating caps for Spain and Portugal. If the deals involved were located in other countries, the tranches we study would no doubt bear distinctly higher ratings.

Figure 2 shows the portfolio breakdown by Weighted Average Life (WAL). The WAL for most tranches is less than 4 years. Three quarters of tranches have WALs less than 6 years. Perhaps the most representative tranche in the portfolio has a par value of EUR 50 million, a rating of BBB, and a maturity of 3 years.
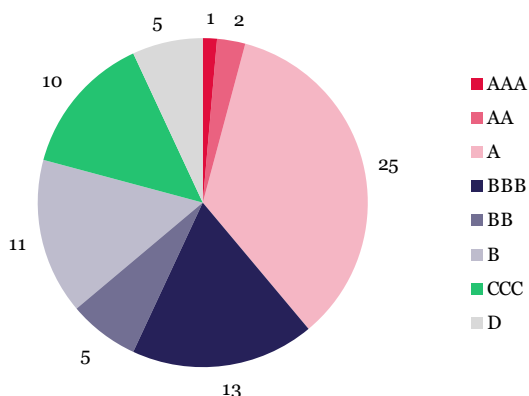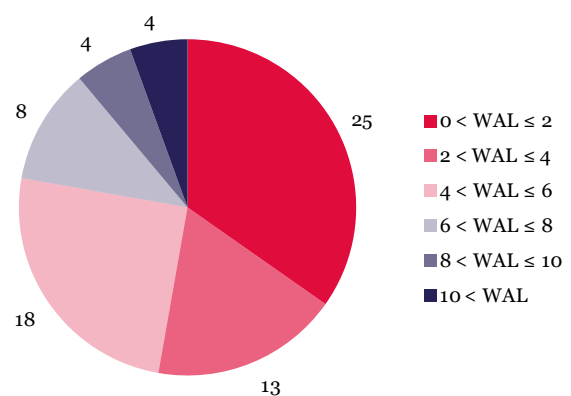
Figure 1: Tranches by rating

Figure 2: Tranches by weighted average life

3. Modelling loan losses

In this and the next section, we describe the methodology we employ to calculate risk statistics for portfolios of securitisation tranches. The methodology involves modelling the stochastic behaviour of loan pools and then building a representation of the cash flow waterfalls on top. To explain the methodology, we describe in this section how loan pools are modelled (and how we calibrate their distributions). In the next section, we describe the approach we take in modelling cash-flow waterfalls.

To model loan pools, we simulate the loss rate, θ_t, through time of a homogeneous pool of underlying loans, t. The methodology employed may be described as follows. Define the transformed loss rate $\tilde{\theta}_t = \Phi^{-1}(\theta_t)$ o a homogeneous loan pool. Here, $\Phi^{-1}(\cdot)$ is the inverse of the standard Gaussian distribution function. Lamb and Perraudin (2008) show that, under suitable assumptions, the transformed loss rate, $\tilde{\theta}_t$, follows the Gaussian autoregressive process:

$$\tilde{\theta}_{t+1} = \sqrt{\beta}\tilde{\theta}_t + \frac{1 - \sqrt{\beta}}{\sqrt{1 - \rho_{Pool}}}\Phi^{-1}(q_t) - \sqrt{\rho}\frac{\sqrt{1 - \beta}}{\sqrt{1 - \rho_{Pool}}}\varepsilon_t \tag{1}$$

Here, q_t is an unconditional probability of default of individual loans (which may vary over time), β and ρ_{Pool} are mean reversion and correlation parameters, and ε_t is a standard Gaussian shock equal to

$$\varepsilon_t = -\sqrt{1 - \eta^2}f_t + \eta\,\zeta_t \tag{2}$$

The random variables f_t and ζ_t are standard Gaussian shocks, with f_t being a factor specific to the country and industry of the securitisation pool loans, while ζ_t is an idiosyncratic shock specific to the securitisation in question. η is a parameter describing the weight of the idiosyncratic factor.

To calibrate these processes for each securitisation asset pool, we take the following approach. Within the *Risk Controller™* software, users supply parameters for each securitisation. These parameters include an initial loss rate, θ_0, and the β, ρ_{Pool} and η parameters. Users also input cumulative default rates and spreads, for maturities of 0 to 30 years which are used to calculate the unconditional pool probability of default at time t. The factor shock f_t is assumed to equal a weighted sum of sector and country factors with user supplied weights.

Lamb and Perraudin (2008) provide an estimate of 0.91 for the value of β for corporate and industrial loans. This is based on aggregate data. Autocorrelation for individual securitisation pool loss rates are probably lower so we opt for a value of 0.8. The cumulative default rates used are those provided in Table 24 in Vazza et al. (2014). These go up to a time horizon of 15 years which is sufficient given the maturities of the exposures we study here. The factor shock is calculated based on a single country factor.

We choose the parameters ρ_{Pool} and η based on values suggested in Duponcheele et al. (2013). In this paper the authors consider a granular securitisation pool, and suppose that a default for the ith loan depends on the value of a latent variable, Z_i, which is assumed to satisfy the factor structure

$$Z_i = \sqrt{\rho_{Pool}}Y_S + \sqrt{1 - \rho_{Pool}}\varepsilon_i \tag{3}$$

Here, Y_S and ε_i are standard Gaussian shocks, with Y_S being a factor common to all exposures in the pool. Y_S, in turn, exhibits the following factor structure:

$$Y_S = \frac{\sqrt{\rho}}{\sqrt{\rho_{Pool}}}Y_B + \frac{\sqrt{1 - \rho}\sqrt{\rho^*}}{\sqrt{\rho_{Pool}}}X \tag{4}$$

Here, Y_B and X are standard Gaussian shocks. Y_B is the factor common to all the exposures in the bank portfolio and X is a factor orthogonal to Y_B. In order that Y_S be standard Gaussian, we set $\rho_{Pool} = \rho + (1 - \rho)\rho^*$. The authors recommend using the values $\rho = 0.2$ and $\rho^* = 0.1$, giving a value of 0.28 for ρ_{Pool}. η is analogous to the coefficient of X in (4). This is approximately equal to 0.5.

Once loss rates $\theta_1, \ldots, \theta_T$ for the loan pool have been calculated, the price V_t of the pool at time t may be calculated using the formula

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$$V_t = \sum_{i=1}^{T} c \exp(-r_{t,t+i}i) \left(\prod_{j=1}^{i}(1 - \theta_j) \right) + Q \exp(-r_{t,t+T}T) \left(\prod_{j=1}^{T}(1 - \theta_j) \right) \quad (5)$$

Here, T is the time to maturity, c is the coupon rate, Q is the principal, and $r_{t,t+i}$ is the i-period interest from time t.

4. Modelling cash flow waterfalls

Having constructed a model of loan pools, one may construct a representation of the securitisation cash-flow waterfalls. Cash payments to the holders of a securitisation tranche are determined by a set of rules collectively referred to as a cash-flow waterfall. These rules describe how income on the loan pool including coupon and principal repayments and recoveries in the event of defaults are allocated to the holders of tranches of notes or bonds enjoying different levels of seniority. Contractual coupon and principal payments to the most senior tranches are paid first. Remaining monies are used to meet contractual liabilities to other tranches in order of seniority.

Given a set of cash flows and rules on the rights of different tranche holders, one may straightforwardly allocate payments among the various tranches. Hence, if one simulates pool cash flows using the approach described in the last section, by implementing the cash flow waterfall rules numerically, one may also simulate the cash flows to the tranche holders.

If one wishes to calculate risk statistics (like Value at Risk (VaR) or Expected Shortfall (ES)) for a portfolio of securitisation tranches over a holding period that exceeds the final maturity of the longest dated securitisation, then simulating the loan exposures and then using the cash flow waterfall rules to simulate the tranche cash flows is sufficient to estimate, via Monte Carlo methods, the distribution of tranche payoffs. From this risk statistics like VaRs and risk return trade-offs may be estimated.

More commonly, however, one wishes to calculate VaRs using a holding period shorter than the maturities of the securitisations in question. In this case, one may simulate cash-flows up to the investment horizon and add these cash-flows (discounted up) to the prices of the tranches at the VaR horizon. But, to do this requires that one be able to value the tranche at the horizon in question, which is generally difficult except using a numerical routine like an embedded Monte Carlo. Such embedded or nested Monte Carlo exercises are numerically infeasible since they involve a very substantial computational cost.

To avoid this difficulty of implementing nested Monte Carlos, we use a conditional regression approach similar to that used by Longstaff and Schwartz in the context of American option valuation. To be more precise, to calculate risk statistics for a securitisation with maturity T and J tranches, *Risk Controller*[TM] follows the following steps:

1. A grid of plausible transformed loss rates at the VaR horizon, t_1, is constructed. For each grid node, loss rates are simulated forward until maturity, and this process is repeated N times.

2. Let $c_{j,t}^{(n)}$ be the cash flow at time t, on the jth tranche and on the nth simulation. The summed discounted cash flow at t_1, on tranche j and simulation m, is denoted $DCF_{t,j,t_1}^{(n)}$, and given by the formula

    $$DCF_{t,j,t_1}^{(n)} = \sum_{i=t_1+1}^{T} c_{j,i}^{(n)} P_{t,t_1,i} \quad (6)$$

    Here, $P_{t,t_1,i}$ is the forward discount factor at time t for discounting a cash flow at time i back to time t_1.

3. For $s = 1, \dots, S$ a statistic $h_{t_1,s}^{(n)}$ is defined for the loss rate history up to t_1 for simulation m:

    $$h_{t_1,s}^{(n)} = H_{t_1,s}\big((\theta_t : t = 1, \dots, t_1)\big) \quad (7)$$

By regressing the summed discounted cash flows $DCF_{t,j,t_1}^{(n)}$ on the statistics $h_{t_1,s}^{(n)}$, a pricing function F is obtained, which assigns to every loss rate history up to t_1 cash flows to each of the tranches:

$$(c_{j,t}: j = 1, \dots, J, t = 1, \dots, T) = F\big((\theta_t: t = 1, \dots, t_1)\big) \tag{8}$$

4. A further Monte Carlo is performed to calculate risk statistics at t_1 using the pricing function F.

We use 2,000,000 replications for the latter Monte Carlo. In modelling the portfolio, we assume that the underlying loans are paid off at a constant rate until maturity. For each tranche we have an estimate of the weighted average life (WAL), calculated assuming a constant prepayment rate, and taking into account the payment structure of the securitisation. We choose a value for the maturity of the underlying loans that results in a WAL equal to the sum of the WALs of the tranches, weighted by their value as a fraction of the value of the entire deal. Given the linear amortisation schedule, the maturity should be set to twice this weighted sum. We obtain yield data for the underlying loans from Bloomberg.

5. Results

In this section, we present an analysis of our example SME-loan-backed securitisation portfolio implemented using *Risk Controller*TM. Throughout this section, unless otherwise specified, all VaRs are calculated over a horizon of one year and at a 99.9% confidence level, and are presented as a fraction of the current value.

Figure 3 shows a histogram of the total value of the securitisation portfolio at the one year horizon. Note that the distribution is in monetary terms and not in returns. Its left skewed appearance reflects the significant influence of downside credit risk.

Figure 3: Histogram of portfolio values
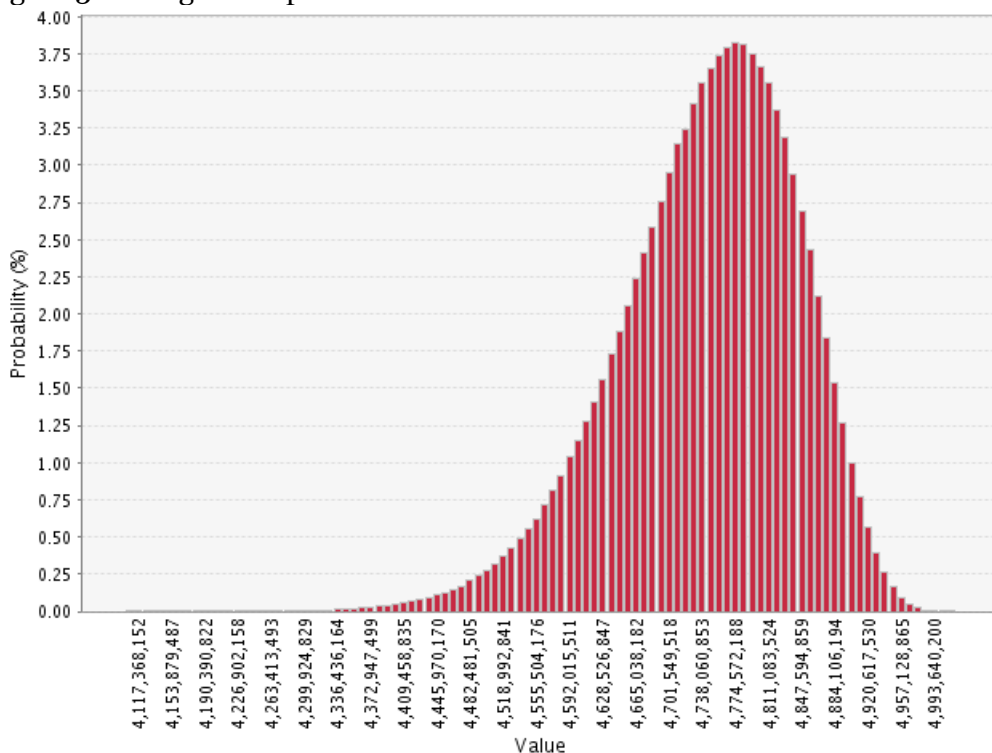
A summary of the results is presented in Table 1. As well as the left skew referred to above, the distribution exhibits slight positive excess kurtosis. The 99.9% VaR as a fraction of the expected forward value of the portfolio is 8.20%, just slightly higher than the Basel I capital percentage of 8%. The Expected Shortfall (ES) is, of course, higher but not very substantially so.

Table 1: Summary of results for Monte Carlo simulation

Risk Measure	Result
Mean value	4,752.48M
Volatility of value	98.90M
Skewness coefficient	-0.62
Kurtosis coefficient	3.50
Value-at-risk 99% - % of mean	5.73
Value-at-risk 99.5% - % of mean	6.50
Value-at-risk 99.8% - % of mean	7.50
Value-at-risk 99.9% - % of mean	8.20
Value-at-risk 99%	272.40M
Value-at-risk 99.5%	309.13M
Value-at-risk 99.8%	356.38M
Value-at-risk 99.9%	389.55M
Expected shortfall 99%	323.90M
Expected shortfall 99.5%	358.88M
Expected shortfall 99.8%	403.19M
Expected shortfall 99.9%	435.50M

Note: The table presents summary statistics for the portfolio, calculated using Monte Carlos. All figures are given in Euros and are discounted to the present.

We now consider risk statistics for individual exposures considered as part of the wider portfolio. Figure 4 shows the 20 exposures for which the marginal VaRs are largest. The largest MVaR is €84 million, more than 20% of the total portfolio VaR. The top two exposures are both from BBVA-originated deals, and were assigned credit ratings of B- and BBB respectively. After the top ten MVaRs, the risk on individual exposure is quite low.

Figure 4: The 20 largest one year marginal VaRs
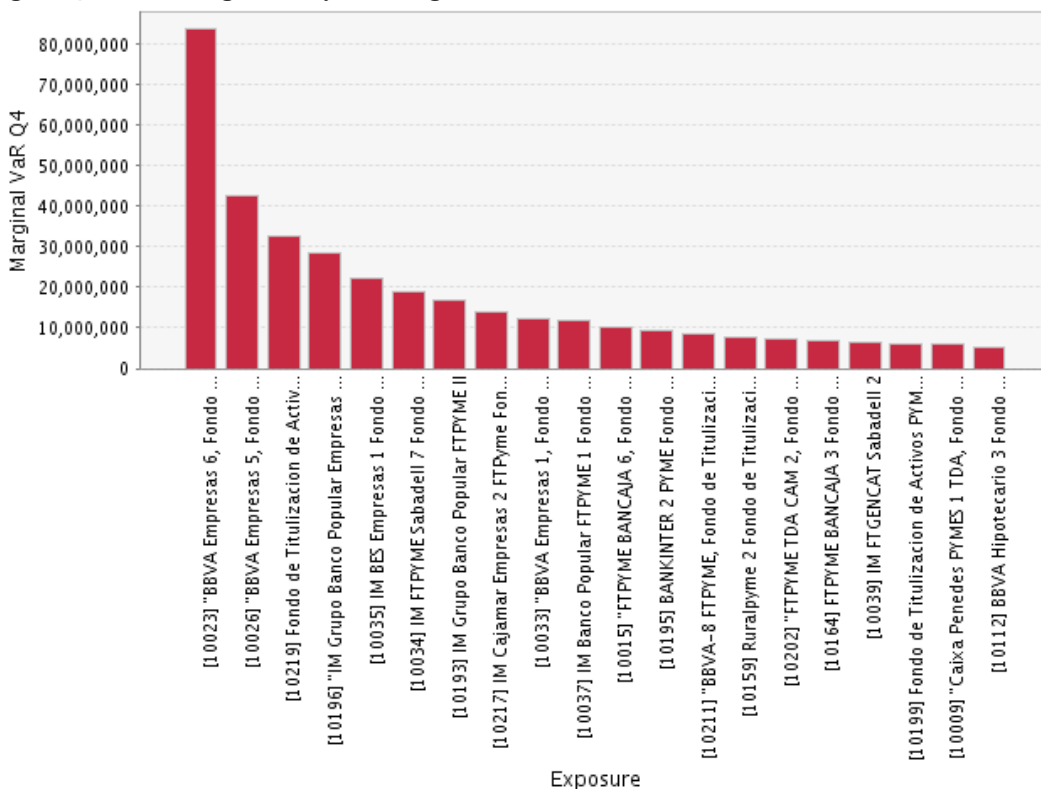
We now examine how the MVaRs are related to different tranche characteristics to understand the variation in risk across the portfolio. Figure 5 shows the relation between tranche attachment points and their MVaRs. The attachment point of a tranche is defined as the fractional loss on the underlying loans above which the loss is absorbed by the tranche. For tranches with attachment points of 0.3 or above the MVaR is negligible, while tranches with attachment points below 0.2 exhibit high MVaRs. By definition, in order for a tranche with an attachment point if 0.3 or above to sustain losses, the losses on the underlying pool of loans must exceed 30%. The low MVaRs attributed to these tranches reflect the low probability of this eventuality.

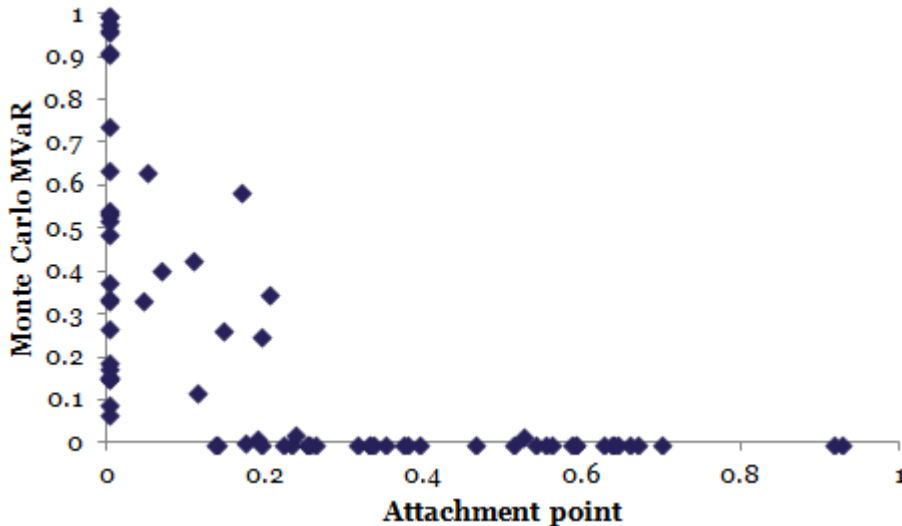Figure 5: Monte Carlo MVaR versus attachment point

Figure 6 shows how the estimated Weighted Average Lives (WALs) of the tranches compare to their MVaRs. Since the more senior tranches are paid off sooner than the less senior tranches, one would expect tranches with low WALs to display lower levels of risk than those with higher WALs.

Figure 6: Monte Carlo MVaR versus Tranche weighted average life (years)
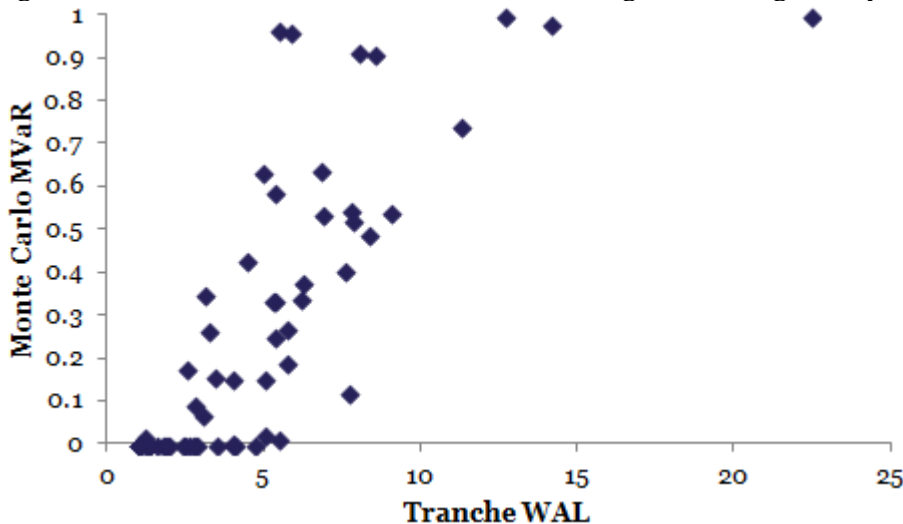

Figure 7 shows how the marginal VaRs of the tranches vary according to rating. Higher grades correspond to uniformly low MVaRs, whereas tranches with lower grades display much more variation. As noted above, the interpretation of rating is complicated in the case of the portfolio we examine here because the ratings agencies employ sovereign rating caps, restricting the rating that a securitisation tranche may attain to some number of rating notches above the rating of the sovereign borrower in its jurisdiction. For Spanish and Portuguese tranches, these caps have been binding. Nevertheless, a clear negative association is apparent from the figure between MVaR and a high rating grade.

Figure 7: Monte Carlo MVaR versus rating modifier
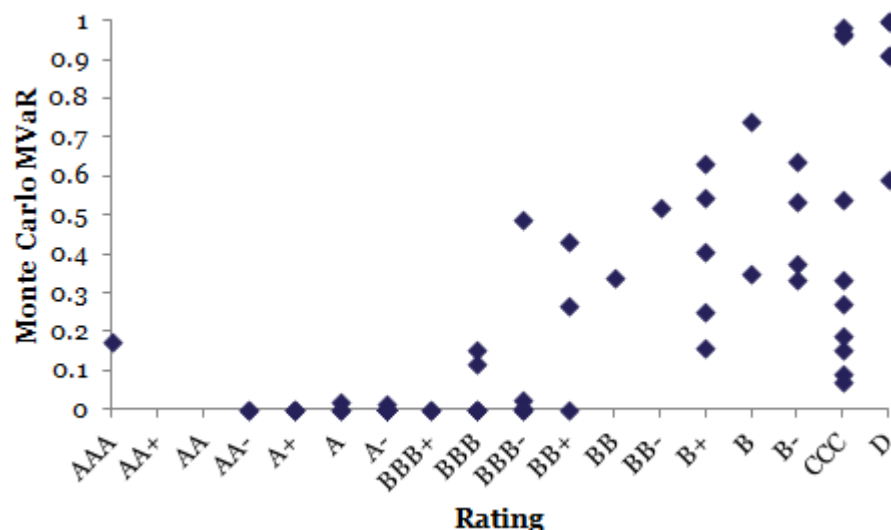
6.   Comparison with AFA

In this section, we compare the marginal VaRs obtained from *Risk Controller™* to those calculated using the Arbitrage Free Approach (AFA) described in Duponcheele et al. (2013). We begin by giving a brief summary of the AFA. The latter is a stylised analytical measure of the marginal VaR of a securitisation exposure suitable for use within a regulatory capital framework.

Here, we employ the AFA as a benchmark for the more realistic (less stylised) approach possible if one employs our Monte Carlo methodology. Specifically, the Monte Carlo approach permits one to use a realistically complicated representation of the cash flows on pool assets and securitisation tranches. It also permits one, if one wishes, to use a more general factor and hence correlation structure for the underlying risks.

Note that the scope to employ a more general factor structure is slightly less important when one is examining a portfolio comprising a single asset class (here, SME-loan-backed deals) within effectively a single geographical region (here Spain and Portugal). With more general portfolios (for example, comprising Spanish and UK deals), a richer correlation structure would be appropriate.

We begin by defining the parameters used in the calculation of the AFA. We consider a securitisation tranche with attachment and detachment points A and D. Let M be the maturity of the securitisation, and let pd_M and LGD be the M-year probability of default and the loss given default on the pool assets. We recall from Section 3 that $\rho = 0.2$, $\rho^* = 0.1$ and $\rho_{Pool} = 0.28$. The expected loss for the tranche may be calculated using equations (9)-(13).

$$EL(A, D) = \frac{(1 - A) \times EL_{Senior}(A) - (1 - D) \times EL_{Senior}(D)}{D - A} \tag{9}$$

$$EL_{Senior}(X) = \frac{LGD \times \bar{N}_2 - X \times PD_{Tranche}(X)}{1 - X} \tag{10}$$

$$\bar{N}_2 = N_2\left(N^{-1}(PD_M), N^{-1}(PD_{Tranche}(X)), \sqrt{\rho_{Pool}}\right) \tag{11}$$

$$PD_{Tranche}(X) = N\left(\frac{N^{-1}(PD_M) - N^{-1}\left(\frac{X}{LGD}\right)\sqrt{1 - \rho_{Pool}}}{\sqrt{\rho_{Pool}}}\right) \tag{12}$$

$$PD_M = N\left(N^{-1}(pd_M) + \frac{(M - 1)\gamma}{\sqrt{M}}\right) \tag{13}$$

γ is a risk-premium parameter, which, according to Bohn (2000), should be assigned a value between 0.3 and 0.5. We use a value of 0.3 in our calculations. N is the normal cumulative distribution function and $N_2(,,\rho)$ are is the bivariate normal cumulative distribution functions, with correlation ρ.

To calculate the marginal VaR using the AFA, we require the stressed M-year probability of default on the pool assets $PD_{\alpha,M}$. Duponcheele et al. (2013) provide two ways of calculating $PD_{\alpha,M}$. According to the first method, $PD_{\alpha,M}$ is calculated using the formula

$$PD_{\alpha,M} = N\left(\frac{N^{-1}(PD_M) - \sqrt{\frac{\rho}{M}}N^{-1}(\alpha)}{\sqrt{1 - \frac{\rho}{M}}}\right) \tag{14}$$

Here, α is the confidence interval, taken here to be 0.001.

In the second method, $PD_{\alpha,M}$ is chosen so as to ensure that the total capital requirement for all the tranches of a securitisation is equal to the IRBA capital requirement for the underlying loans, K_{IRB} (see Basel Committee on Banking Supervision (2006)). This implies that

$$PD_{\alpha,M} = \frac{K_{IRB}}{LGD} + PD_M \tag{15}$$

We replace PD_M by $PD_{\alpha,M}$ and replace ρ_{Pool} with ρ_M^*, where ρ_M^* is given by the formula

$$\rho_M^* = \frac{(1-\rho)\rho^* + (M-1)\rho_{Pool}}{(1-\rho) + (M-1)} \tag{16}$$

We, thus, obtain $MVaR(A,D)$ instead of $EL(A,D)$. For the M-year probability of default pd_M, we use the cumulative default rates provided in Table 24 in Vazza et al. (2014) and linear interpolation.

Figure 8 shows how the marginal VaRs calculated using the Monte Carlo approach compare to the marginal VaRs calculated using the AFA. Here $PD_{\alpha,M}$ is calculated using (14). The linear regression lines and R^2's are shown. The high R^2 may partly be attributed to clustering, with low risk and high risk tranches being assigned MVaRs close to 0 and 1 respectively, regardless of the calculation method, but with the two MVaR calculations differing noticeably for tranches that fall into neither of the above groups.
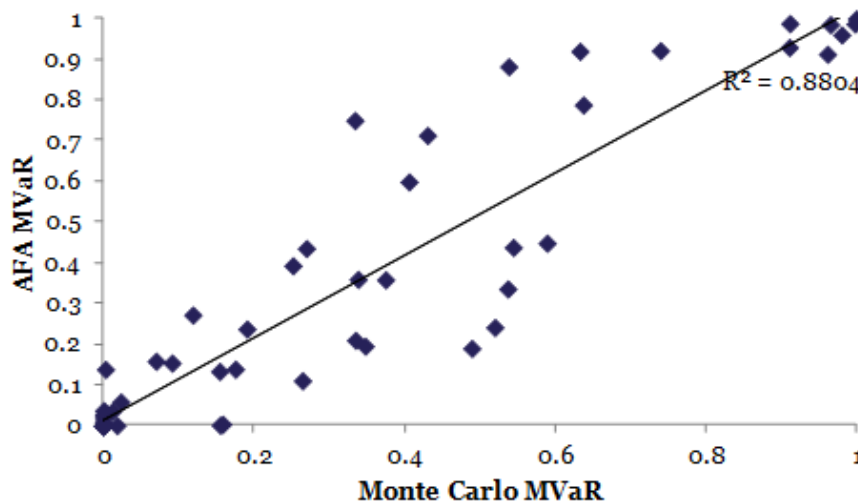
Figure 8: AFA marginal VaR versus Monte Carlo MVaR

Figure 9 shows a similar comparison, with $PD_{\alpha,M}$ calculated using (15). The two sets of results are similar, with (15) producing slightly higher MVaRs.

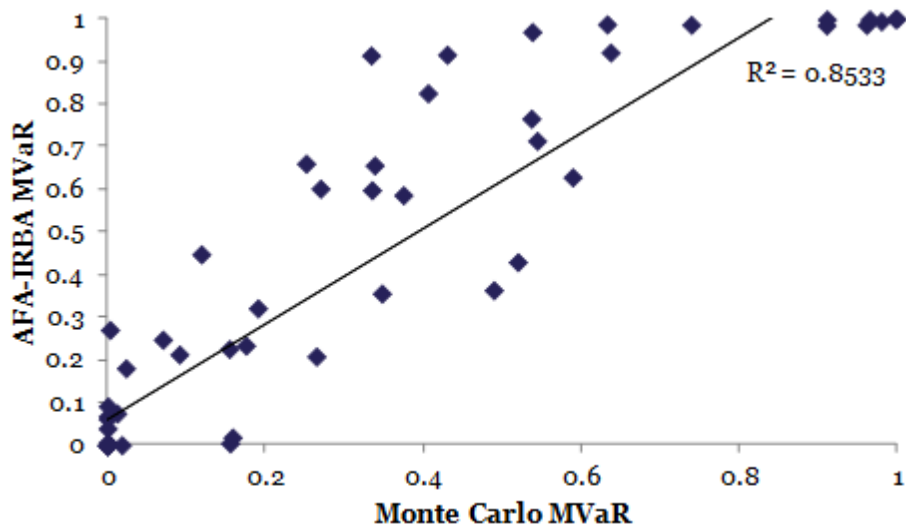Figure 9: AFA marginal VaR with IRBA input versus Monte Carlo MVaR

Table 2 presents the results of regressions of differences between the AFA MVaRs and the Monte Carlo MVaRs (that appear in Figure 8) on different exposure characteristics. The differences depend negatively on the duration of the securitisation and positively on the tranche attachment point. This shows that the AFA over-estimates the contribution to risk of duration and under-estimates the effect of attachment point compared to the more realistic Monte Carlo model. The default profile dummy variable employed in the regression takes a value of unity when the pool default probability profile is riskier and zero otherwise. The coefficient for this variable is positive, as expected, and statistically significant.

Table 2: Regression statistics for Monte Carlo MVaR - AFA MVaR

Variable	Estimate	SE	tStat	pValue
Intercept	0.02	0.05	0.48	0.64
Deal duration	-0.02	0.01	-1.44	0.16
Attachment point	0.02	0.05	0.37	0.71
Default profile	0.09	0.03	2.93	0.00

Note: The table shows the results obtained by regressing the difference between the AFA marginal VaR and the Monte Carlo marginal VaR on the duration of the deal, the attachment point of the tranche and a dummy variable for the default profile. In our analysis two default profiles are used in the Monte Carlo simulation, the default profile variable is set to 1 for exposures to securitisations assigned to the riskier of the two default profiles, and 0 otherwise.

Finally, we compare, in Figures 10 and 11, the Monte Carlo marginal VaRs to SSFA capital charges. The SEC-IRBA and SEC-SA capital charges are calculated according to the rules specified in Basel Committee on Banking Supervision (2014). The SEC-IRBA capital charges are slightly lower than the Monte Carlo MVaRs for some exposures. This reflects, in part, the firm-size adjustment for SMEs used in the IRBA. But, the regulatory and Monte Carlo MVaRs are generally positively correlated.

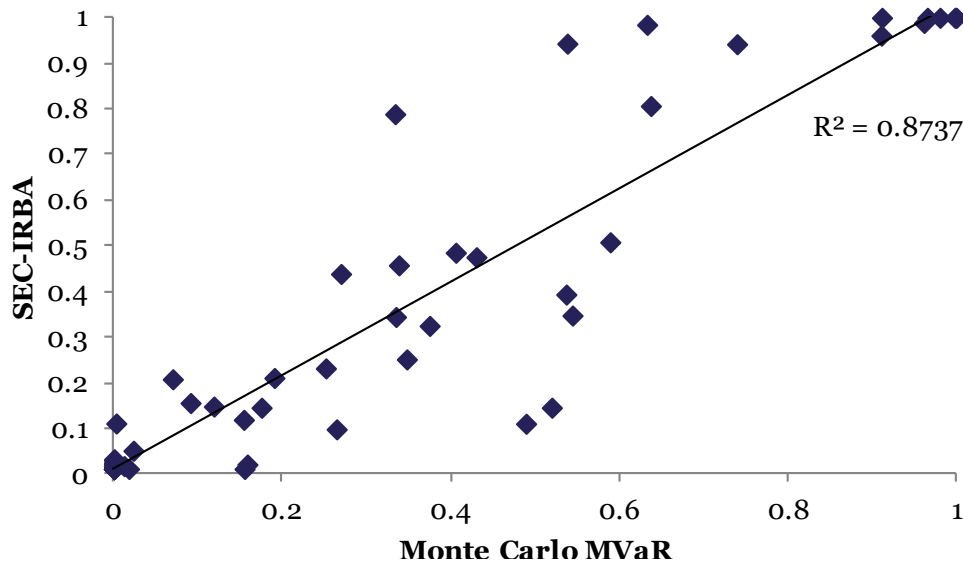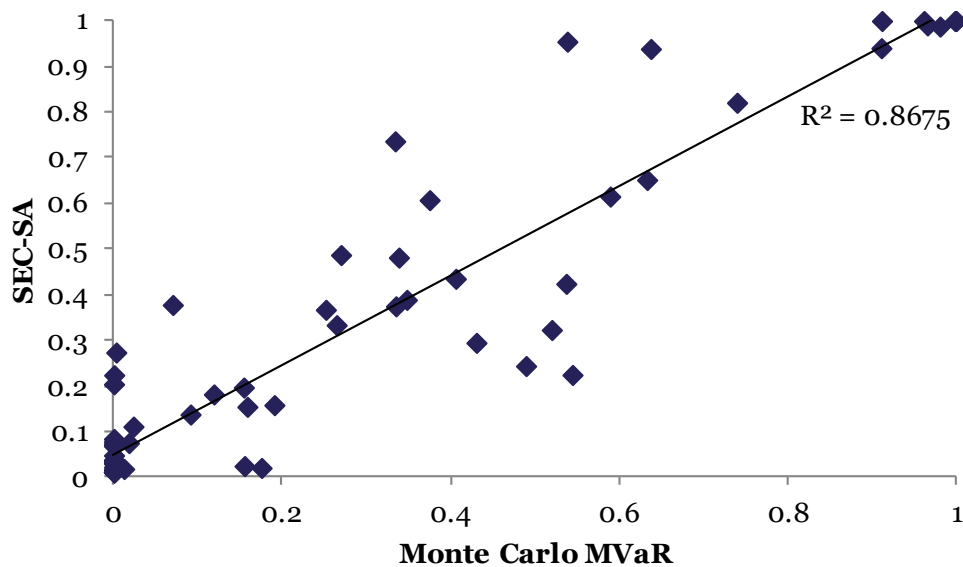Figure 10: SEC-IRBA versus Monte Carlo MVaR

Figure 11: SEC-SA versus Monte Carlo MVaR

7. Conclusion

The models presented in this note constitute a toolbox of rigorous techniques for analysing securitisation portfolio risk. As such, they permit the user to analyse with confidence the risks in holding securitisation exposures, to understand the risk return trade-offs for such exposures and appropriate levels of capital. The models presented include a flexible Monte Carlo-based framework in which multi-period securitisations with complex cash-flow waterfalls may be represented and a simple, stylised analytical model, the Arbitrage Free Approach (AFA) (as proposed by Duponcheele, Perraudin and Totouom-Tangho (2013)).

We show in a case study of Spanish and Portuguese SME-loan-backed securitisation tranches that our Monte Carlo model provides intuitively reasonable risk measures. Using it, we calculate Marginal Value at Risk (MVaR) measures and show that they are correlated across individual exposures with familiar risk drivers such as attachment point, maturity and agency rating. When implemented under comparable assumptions, the Monte Carlo-based capital numbers also exhibit high cross-sectional correlations with those implied by the AFA and with regulatory capital.

Differences between the capital implied by the Monte Carlo and analytical models reflect the more realistic modelling of the cash-flow waterfall possible within the former and differences in the modelling of pool loan defaults. (This modelling is multi-period in the Monte Carlo model and single-period in the stylised models.)

8. References

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